



**Deutsche
Gesellschaft
für Nuklearmedizin
e.V.**

Translational Research in Molecular Imaging and Radionuclid Therapy

August 30 – September 1, 2018

Quantitative Imaging and Tracer Kinetic Modelling

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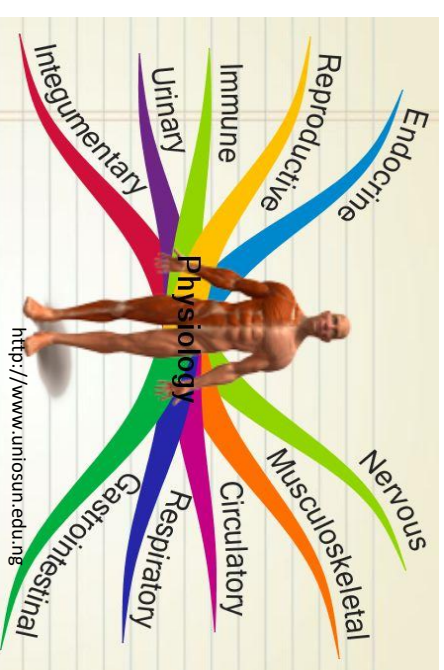
Aim of tracer kinetic modelling is...

in vivo characterization of tissues / organs

with respect to physiological / biophysical parameters

using SPECT / PET imaging

- metabolic rates (micromol substrate / g tissue / min)
- perfusion (ml blood / g tissue / min)
- receptor density (fmol / mg)
- affinity of tracer for target (nM)
- density of pathological targets, e.g. A β -plaques, tau-tangles...



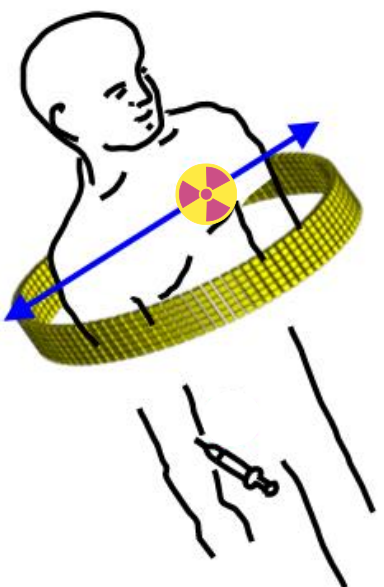
Contents

- basic principle of PET and SPECT radionuclide imaging & tracer principle
- tracer kinetic modelling
 - application in medical research
 - how it works



chironomus plumosus larvae

PET and SPECT radionuclide imaging: basic principle



1.

function /
(patho)physiological process /
target

labeling

2.

relevant molecule /
ligand

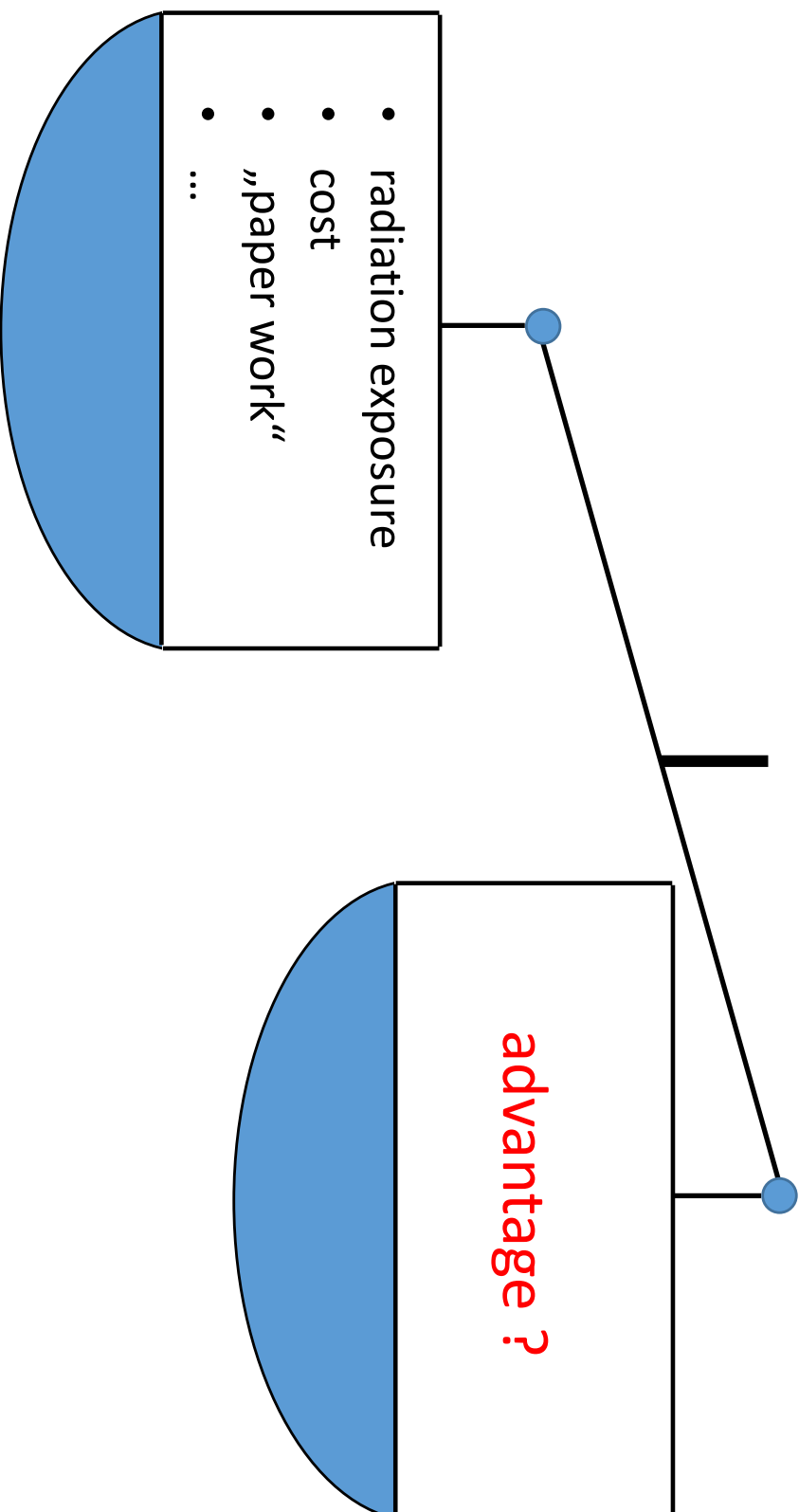
+

radionuclide

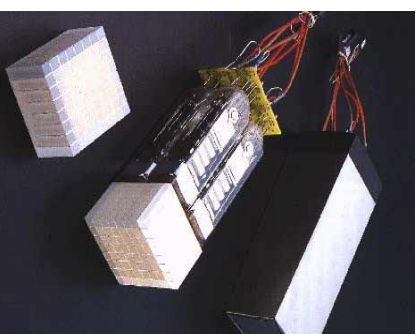
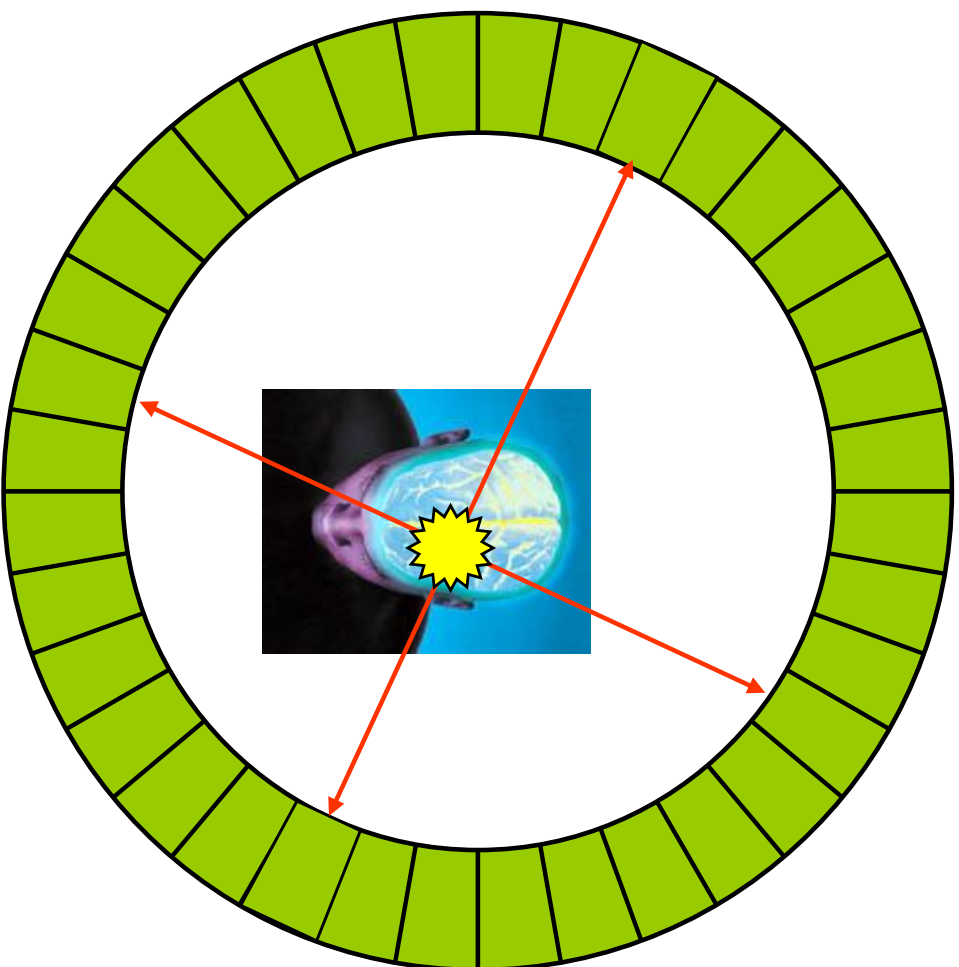
tracer

3.

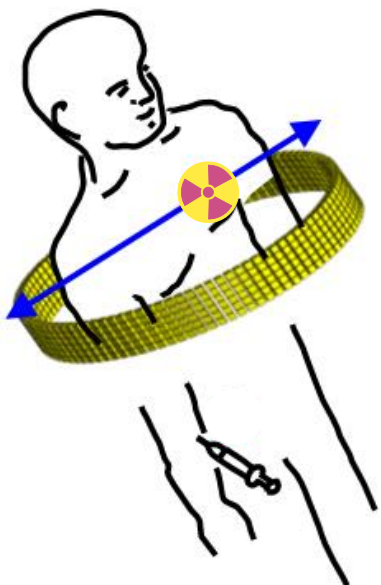
radioactive labeling



PET: high sensitivity for detection of positron decays



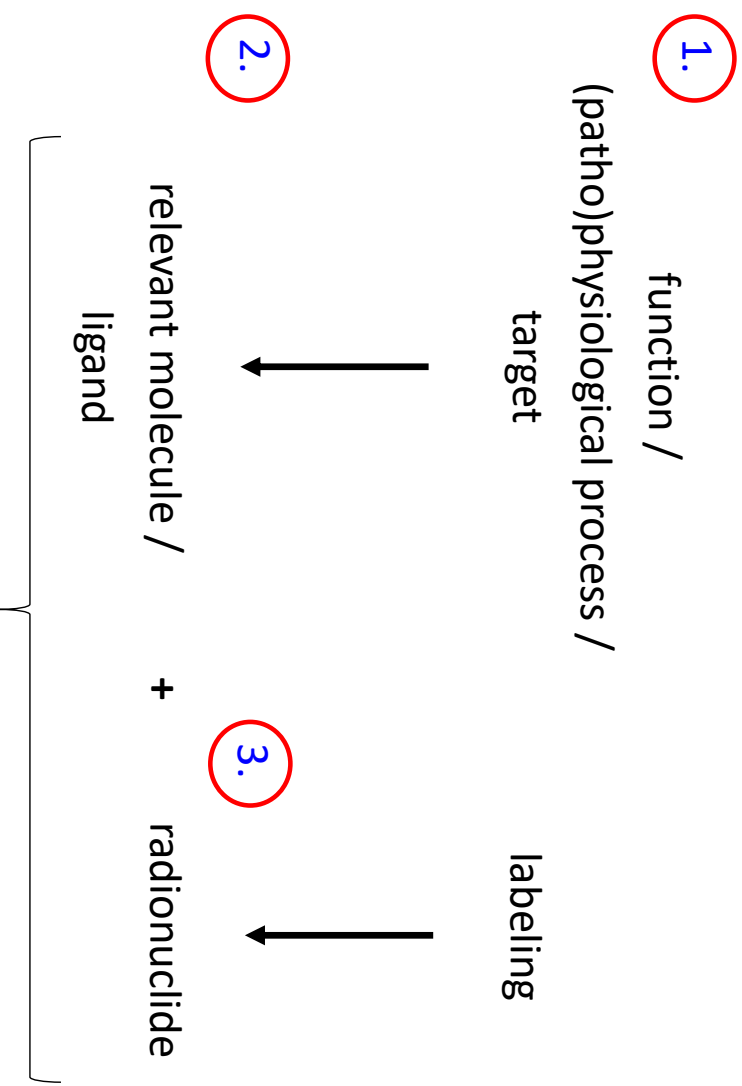
PET and SPECT radionuclide imaging: basic principle



typical mass dose

$\mu\text{g} - \text{ng}$

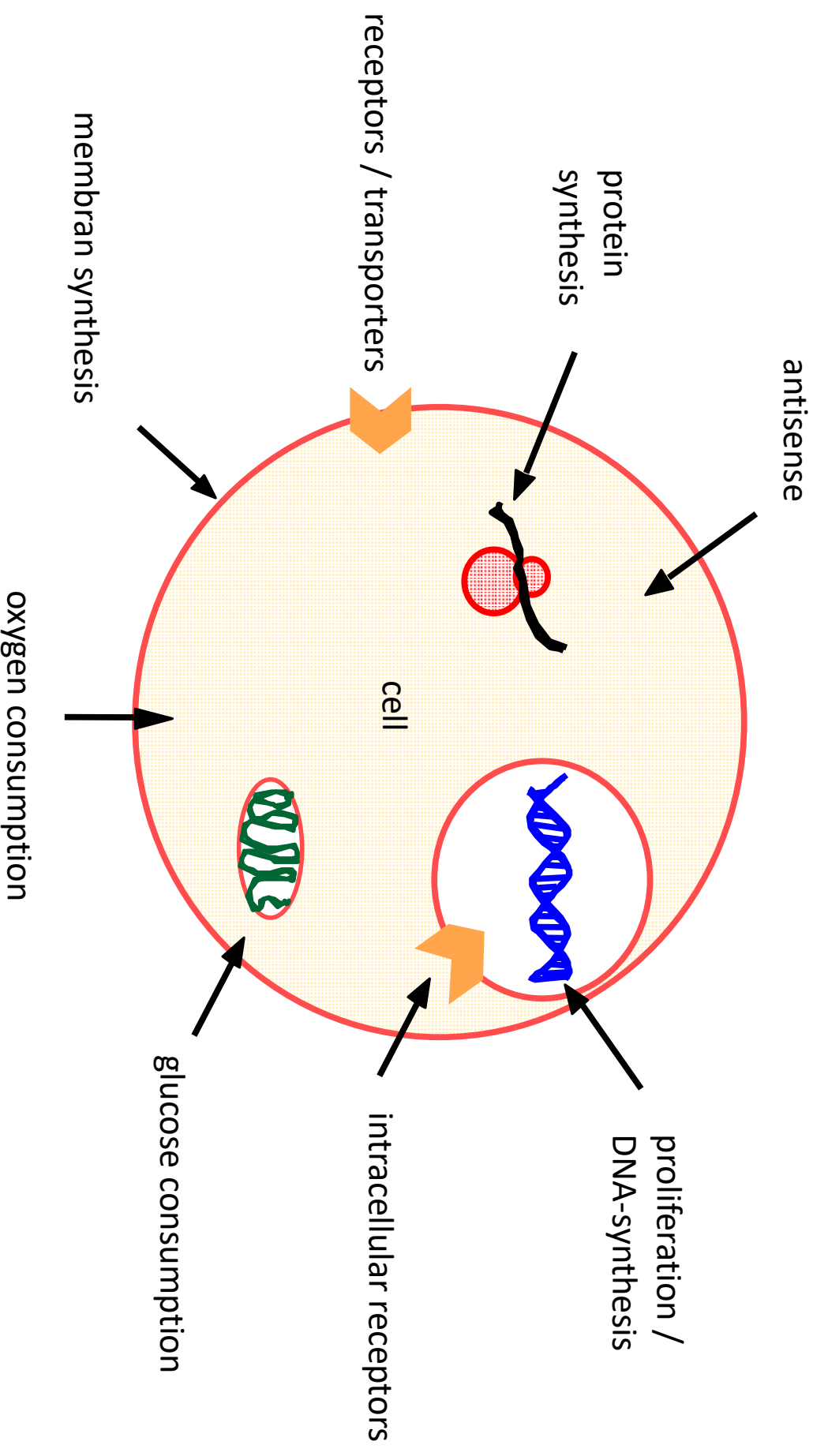
1. no toxic effects → no toxicological restrictions in the selection of relevant molecules



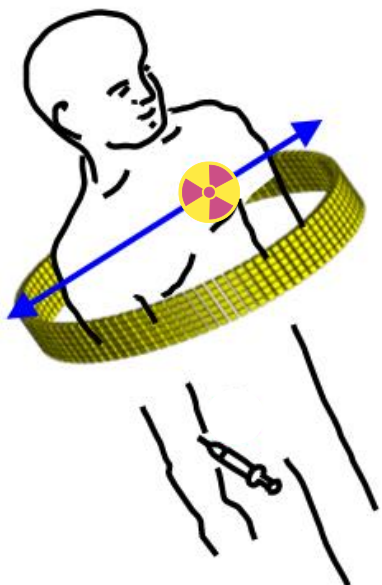
Functions / Targets

Tracer: Iwata, Reference Book for PET Radiopharmaceuticals

Molecular Imaging & Contrast Agent Database (NIH)



PET and SPECT radionuclide imaging: tracer principle

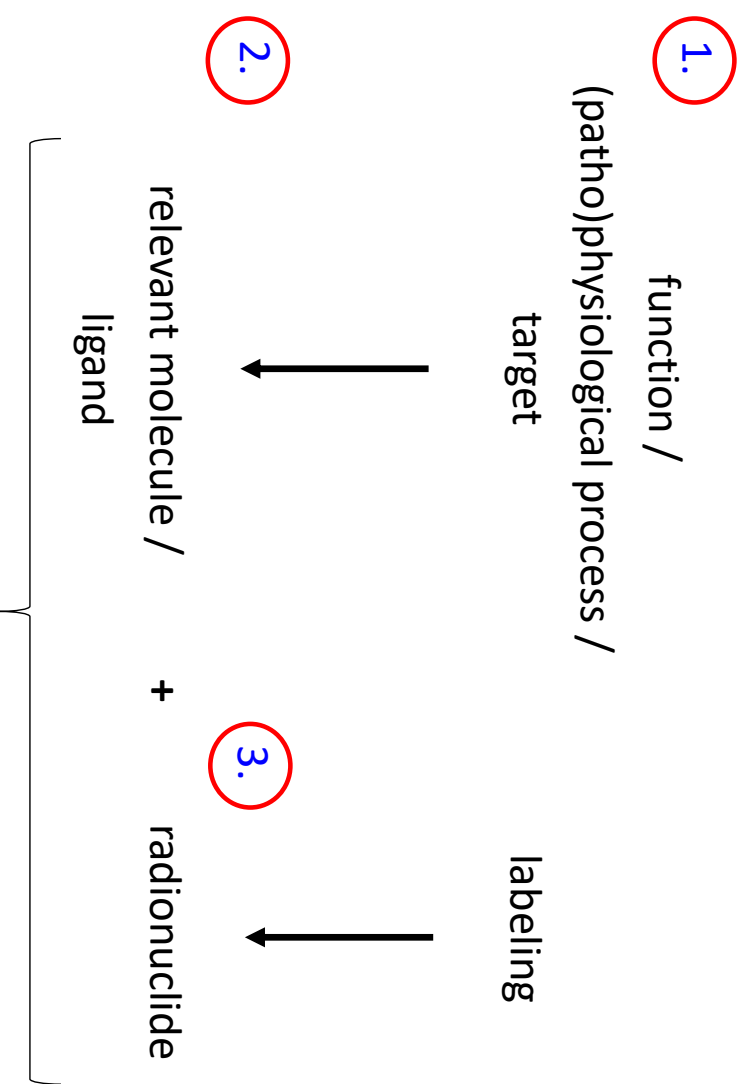


typical mass dose

$\mu\text{g} - \text{ng}$

1. no toxic effects → no toxicological restrictions in the selection of relevant molecules
2. no pharmacological effects → no interference of the tracer with function/target of interest

tracer principle



State of the art PET image quality



2005

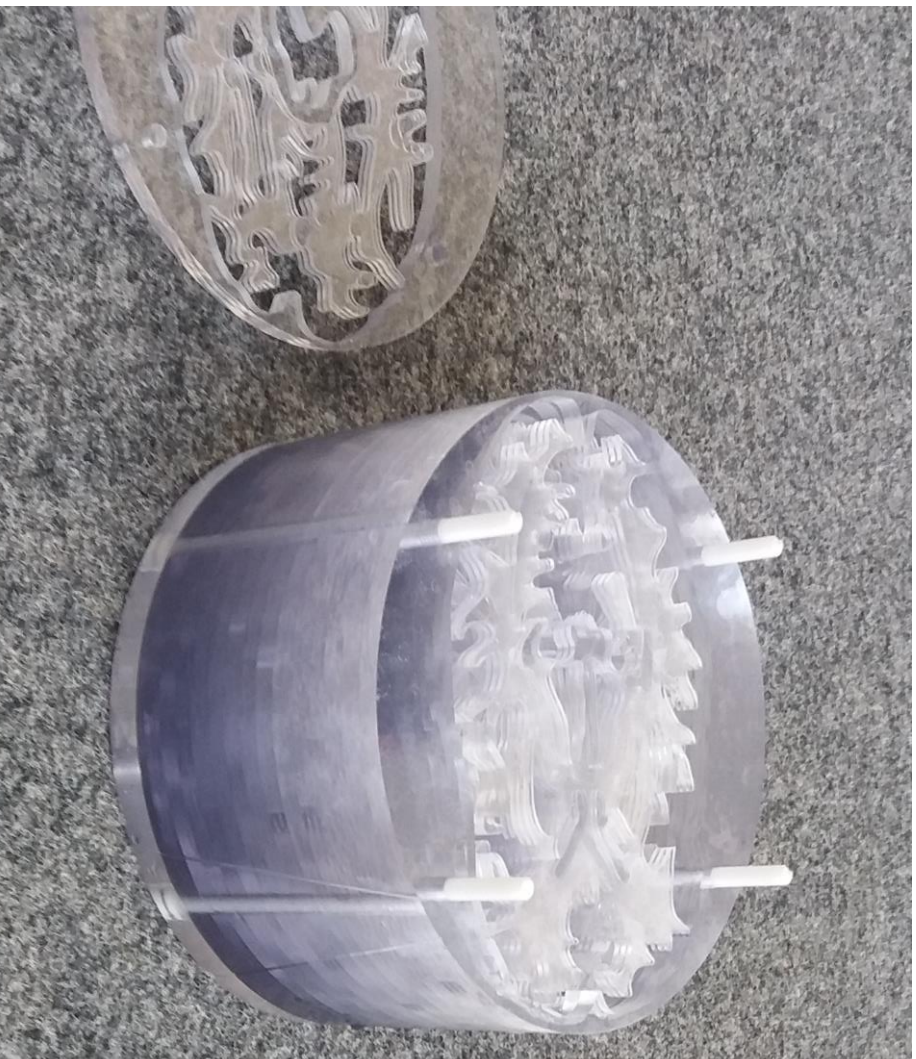


2018

(dsIPM → improved TOF → improved SNR)

Hoffman 3D phantom

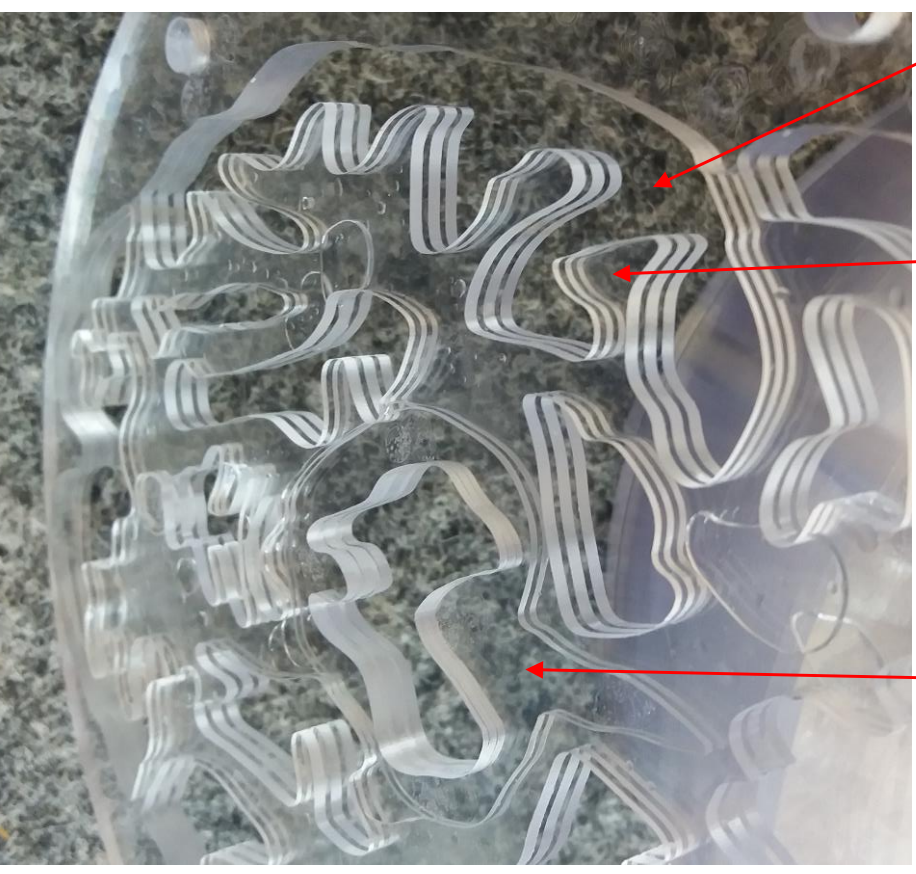
19 acrylic glass inserts made up of 5 slices



gray matter

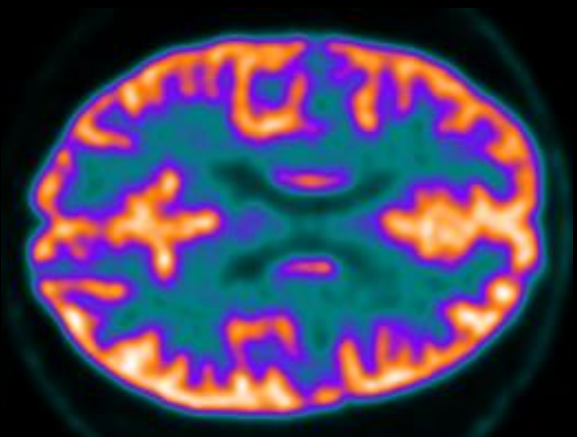
white matter (1:4)

cerebrospinal fluid

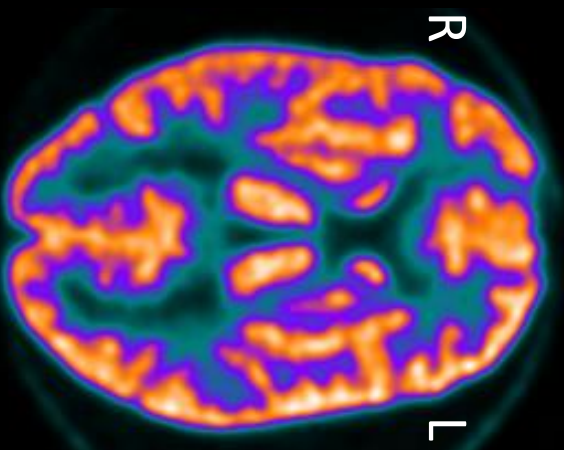


Hoffman 3D phantom

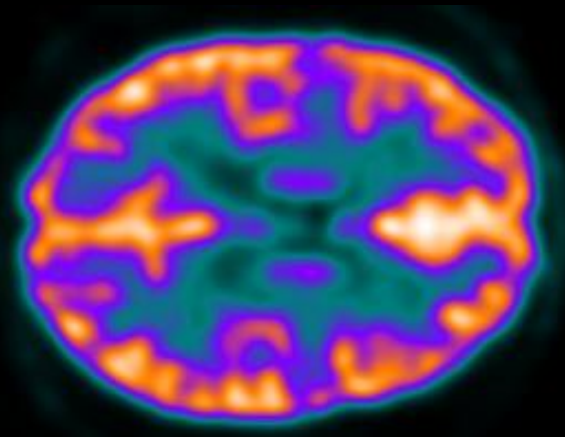
2018



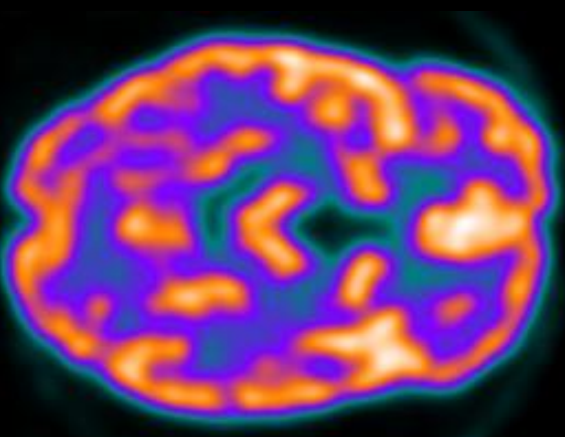
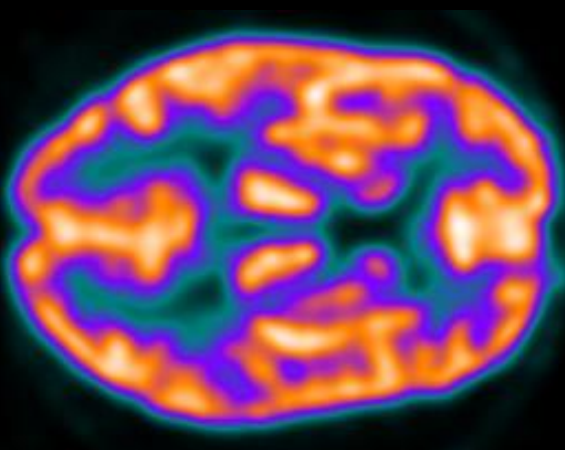
R



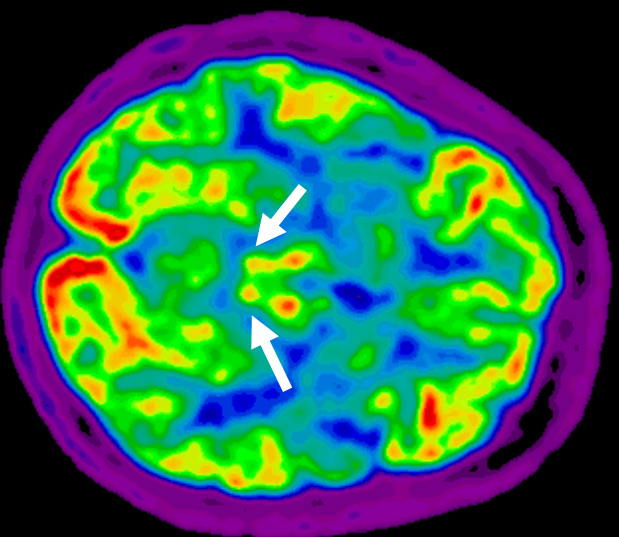
L



2005

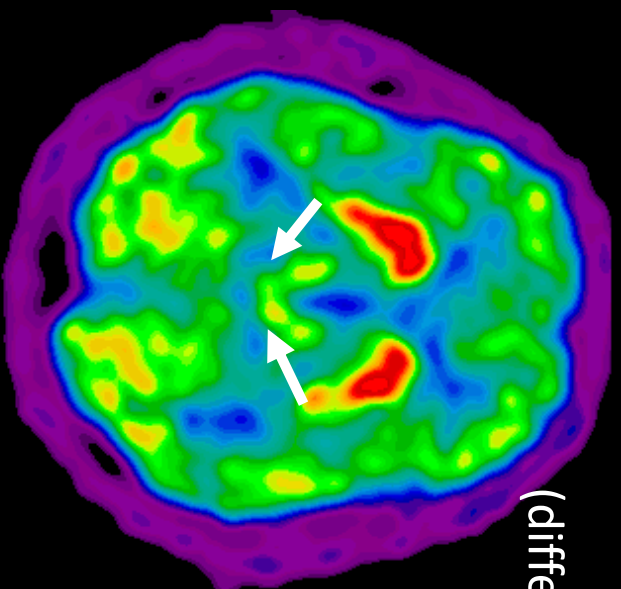
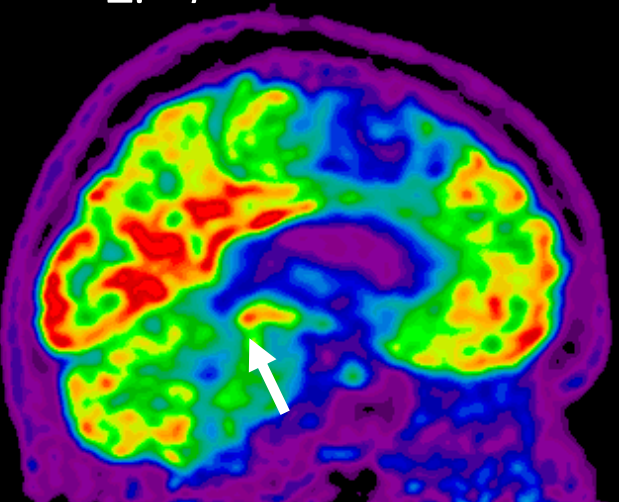


Patient FDG PET scans



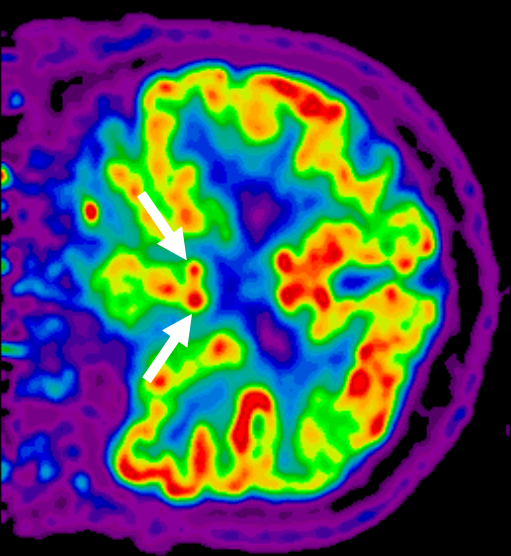
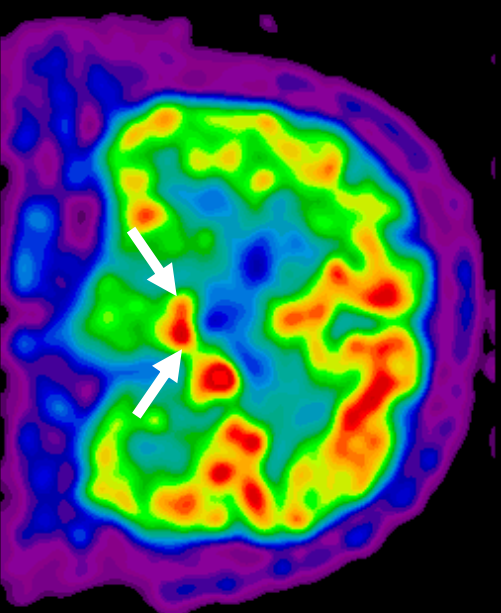
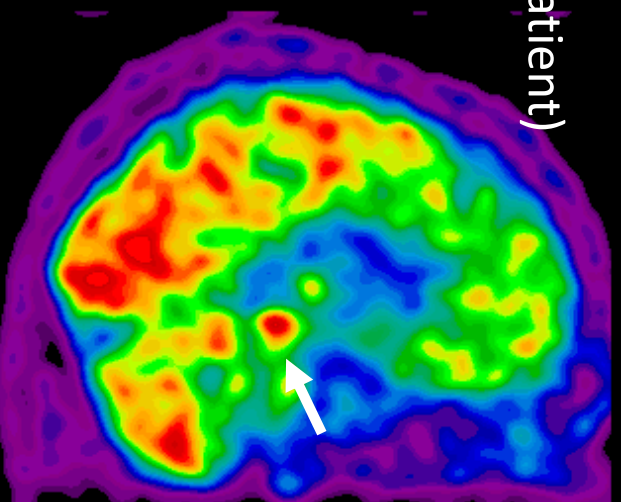
2018

inferior
colliculi

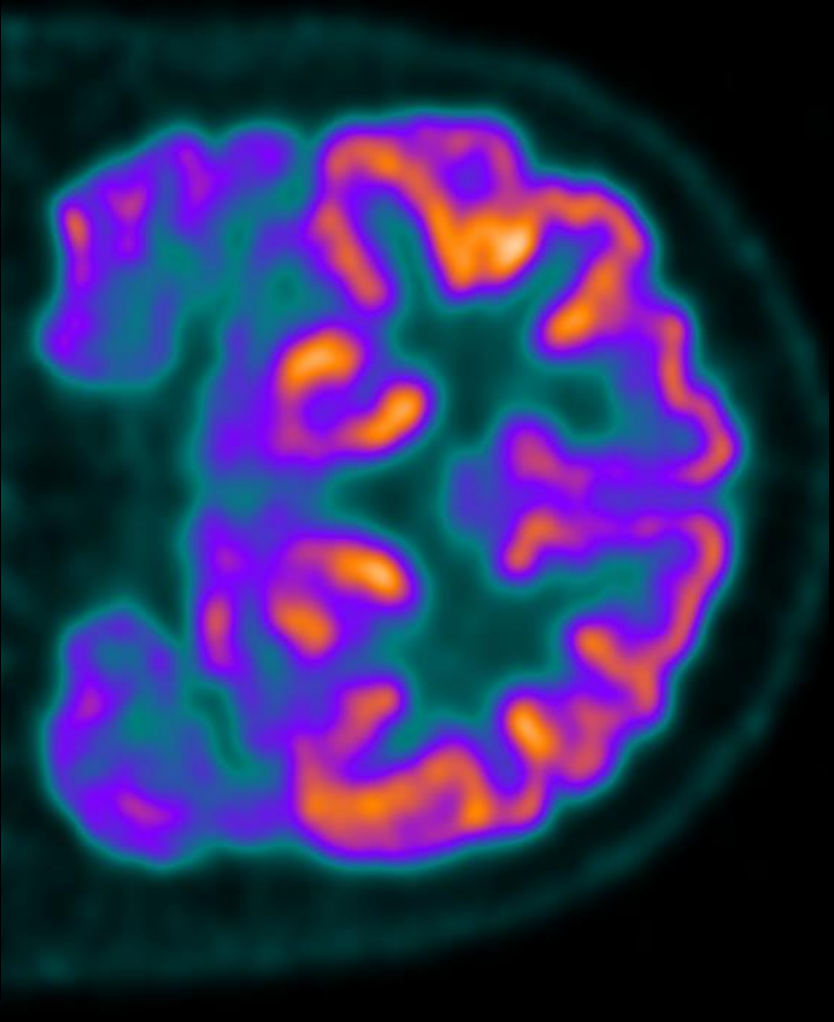
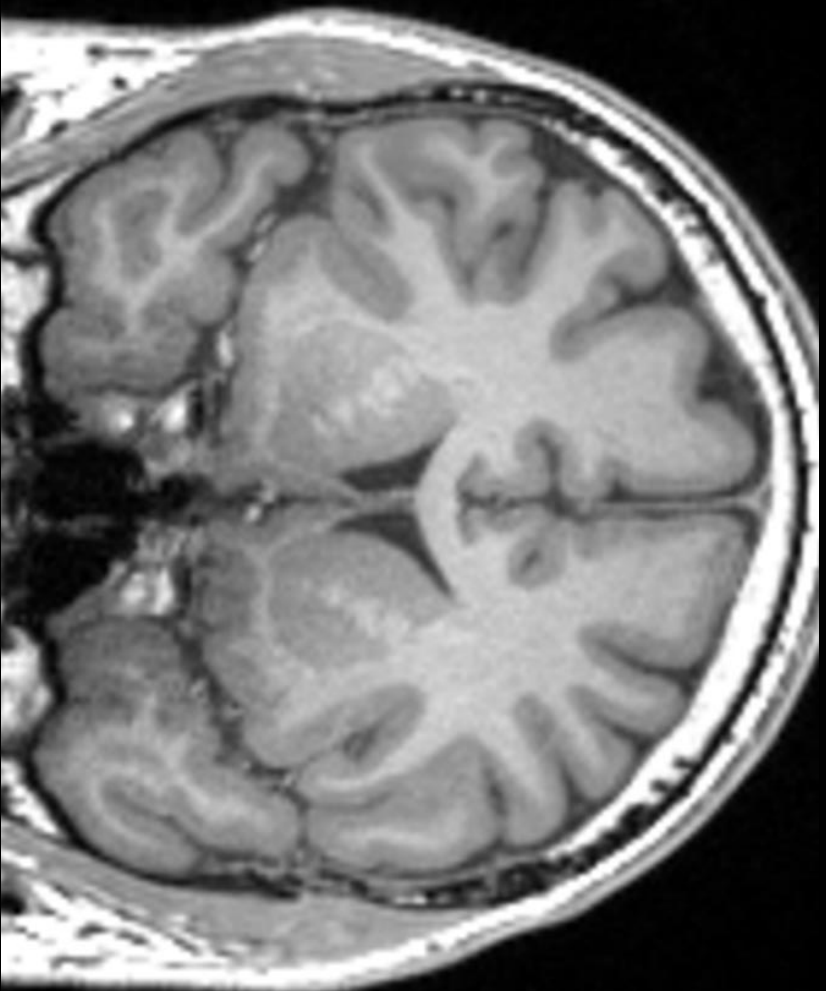


2005

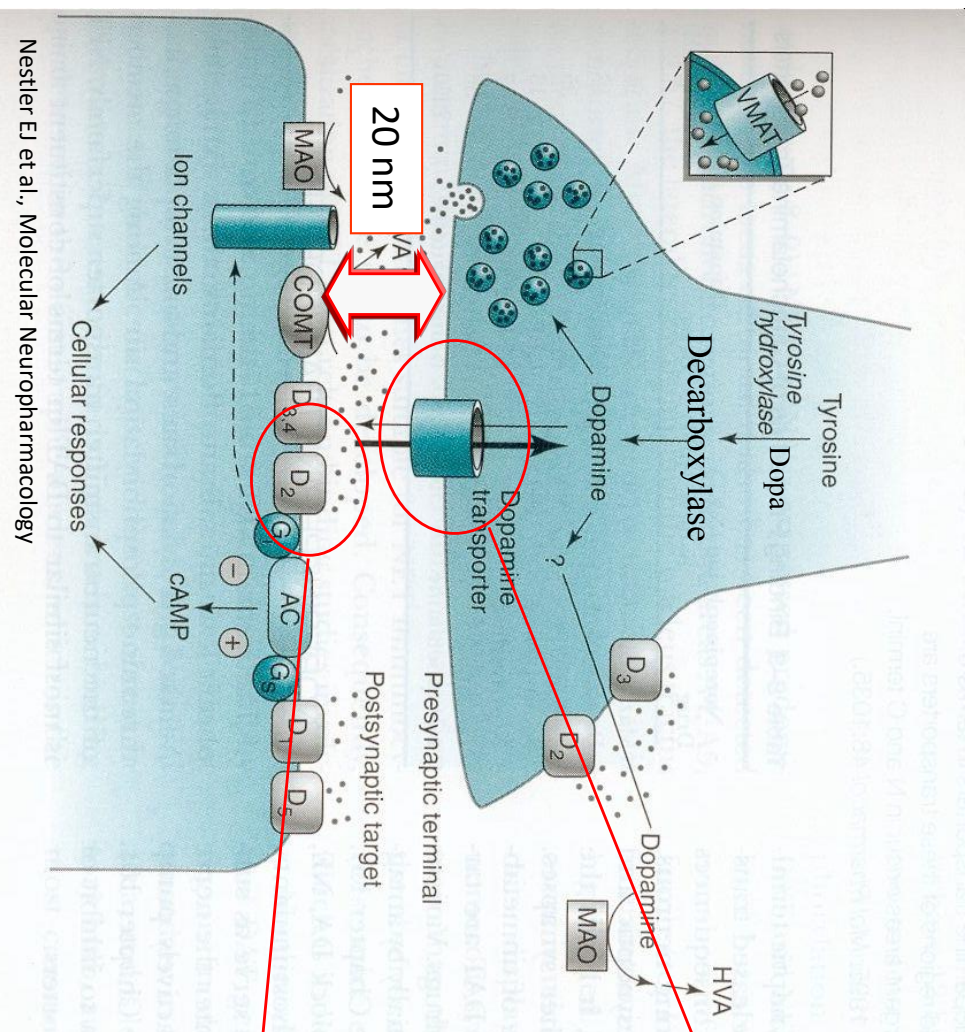
(different patient)



PET vs MRI

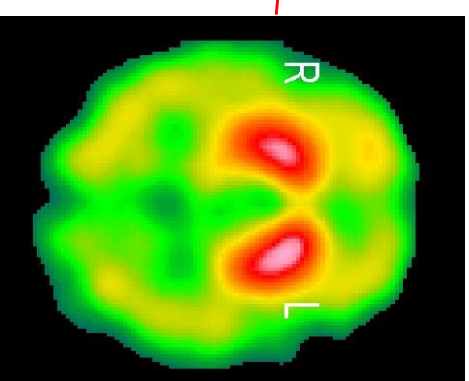
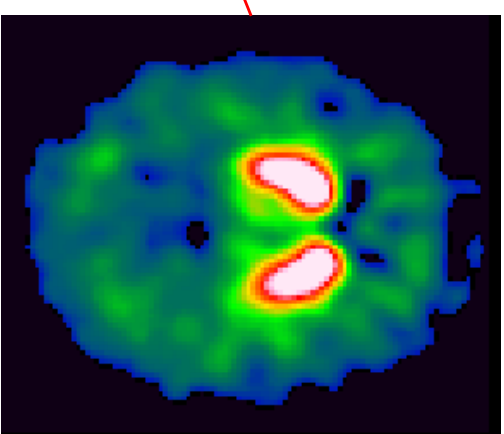


high „intrinsic“ spatial resolution



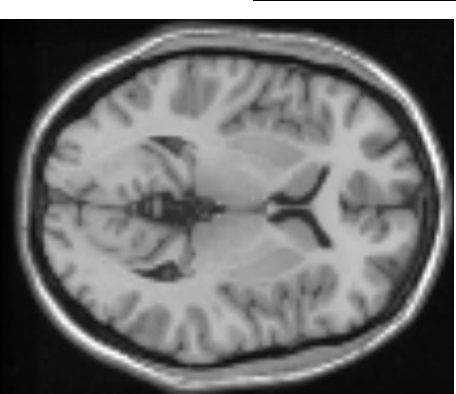
dopamine transporter

I-123-FP-CIT



dopamine D2 receptor

I-123-IBZM



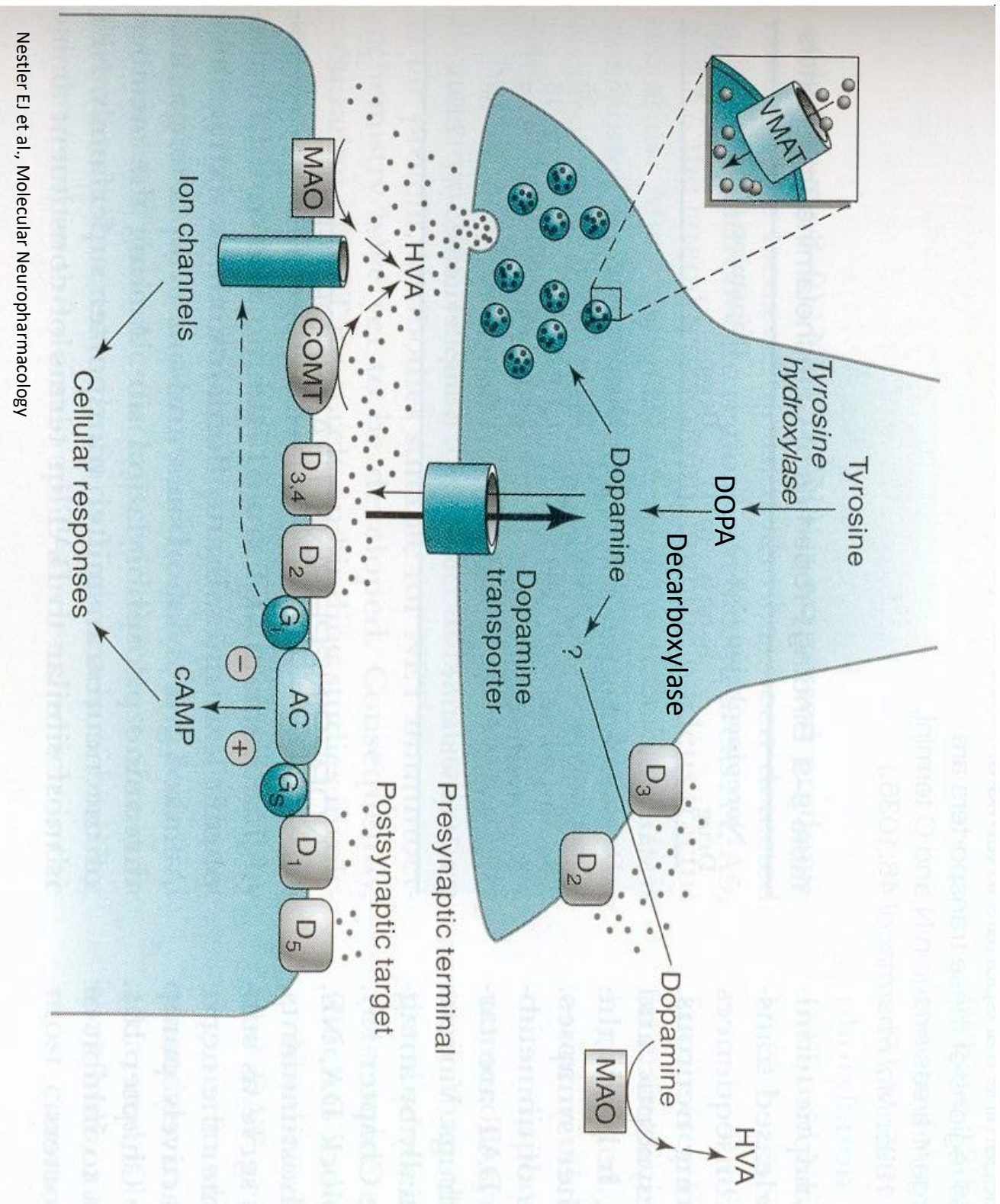
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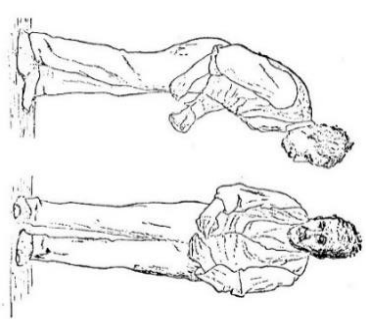
chironomus plumosus larvae

F-18-DOPA PET: dopamine synthesis versus dopamine storage



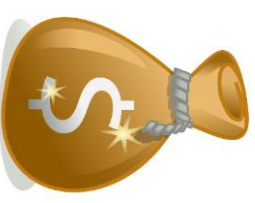
Nestler EJ et al., Molecular Neuropharmacology

Parkinson's disease

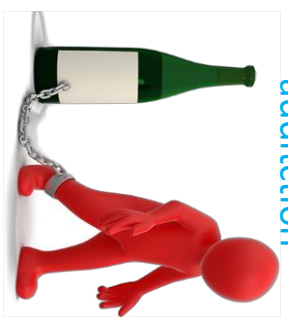


Zeichnung R. Gowers 1886

reward processing



addiction



drugdetails.com

memory



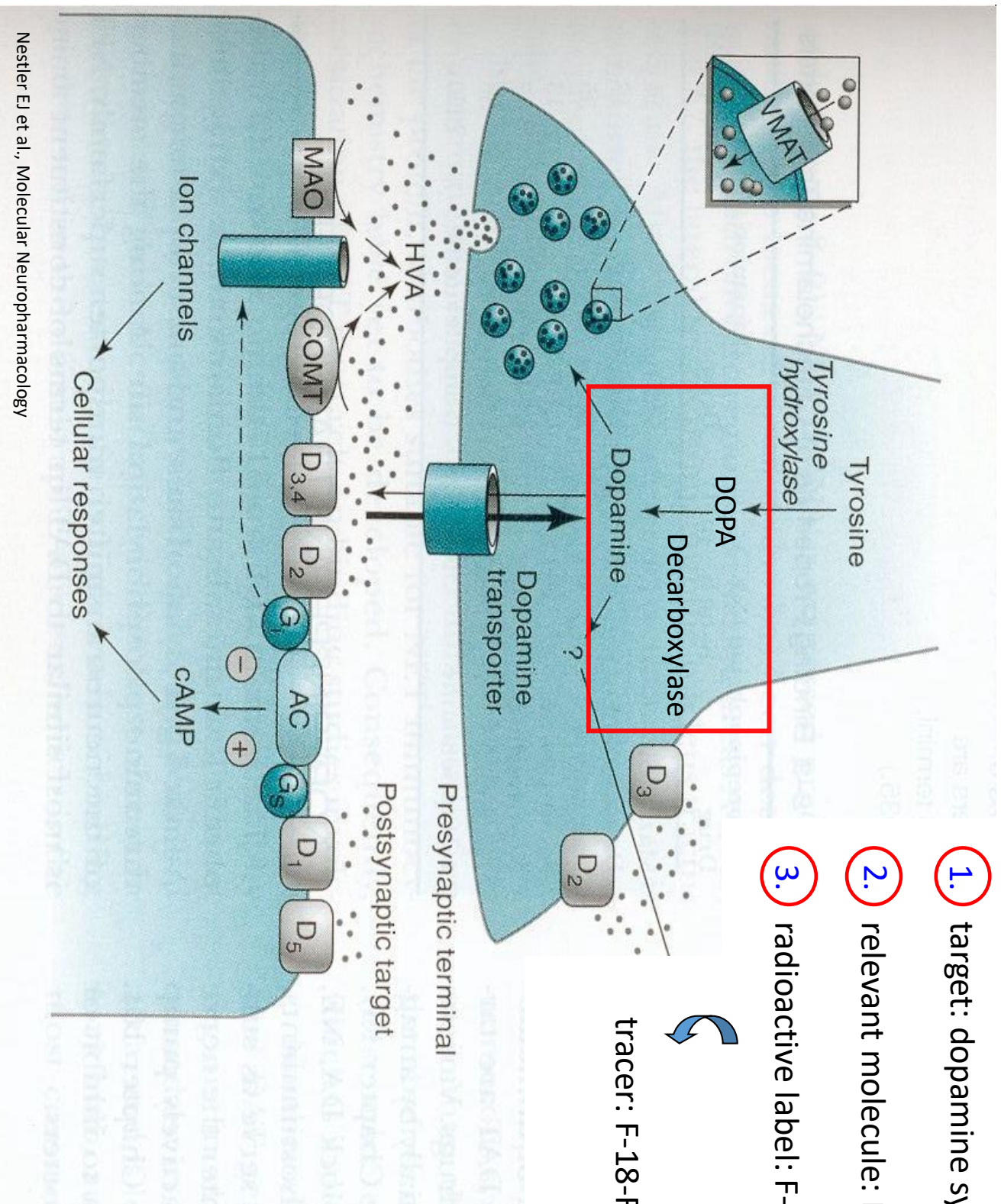
littlethingz.be

F-18-DOPA PET: dopamine synthesis versus dopamine storage

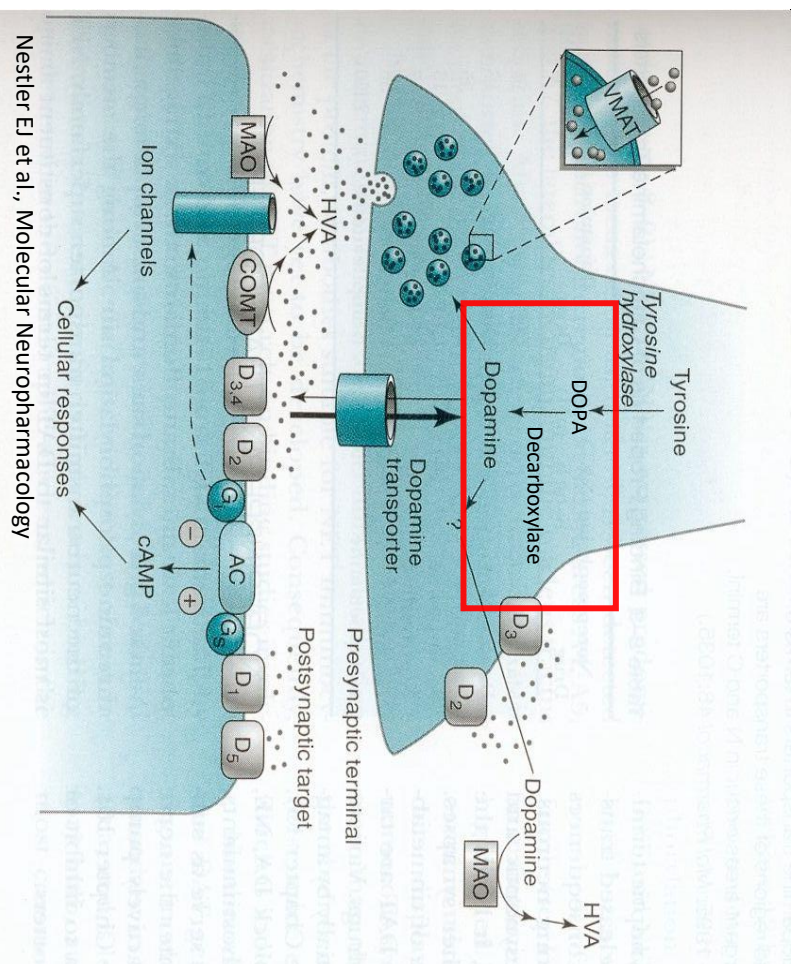
1. target: dopamine synthesis
2. relevant molecule: DOPA
3. radioactive label: F-18



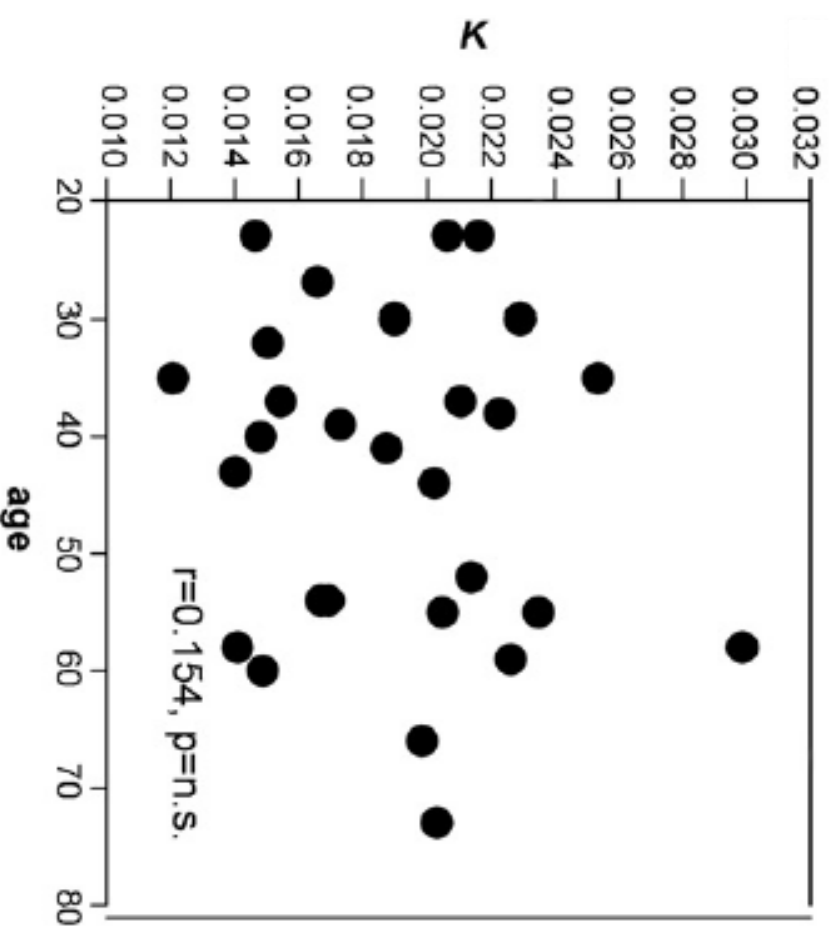
tracer: F-18-Fluoro-L-DOPA (FDOPA)



F-18-DOPA PET: dopamine synthesis versus dopamine storage

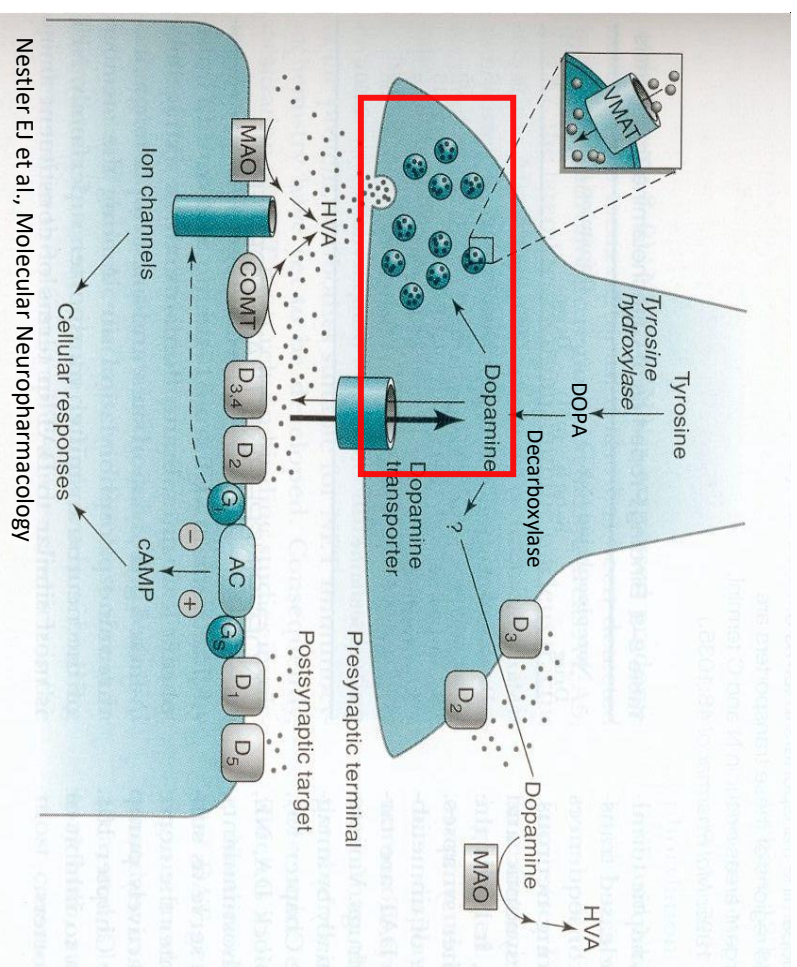


Nestler EJ et al., Molecular Neuropharmacology

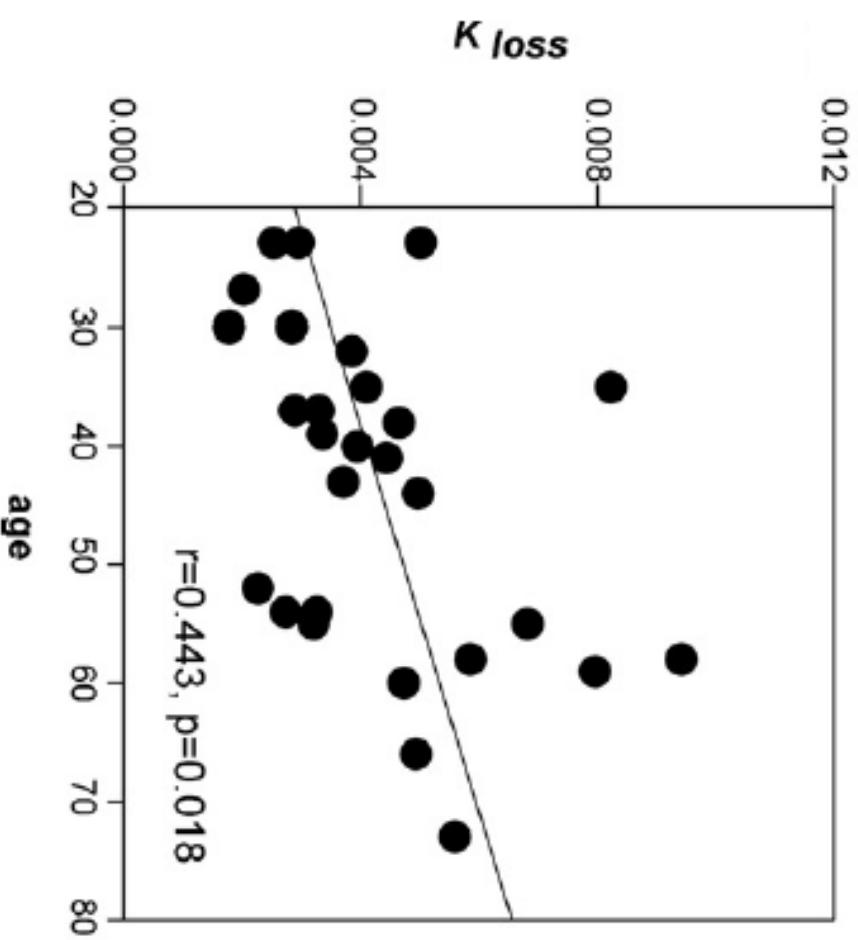


Kumakura et al., Neurobiol of Aging 2010, 31: 447-63

F-18-DOPA PET: dopamine synthesis versus dopamine storage



Nestler EJ et al., Molecular Neuropharmacology



Kumakura et al., Neurobiol of Aging 2010, 31: 447-63

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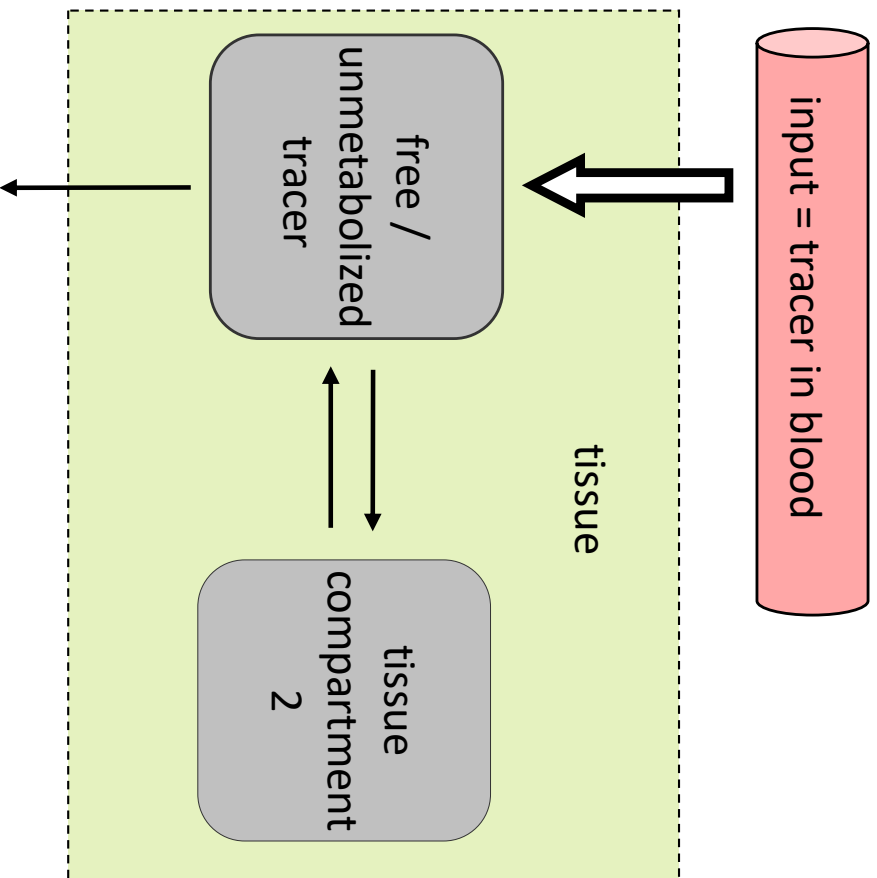
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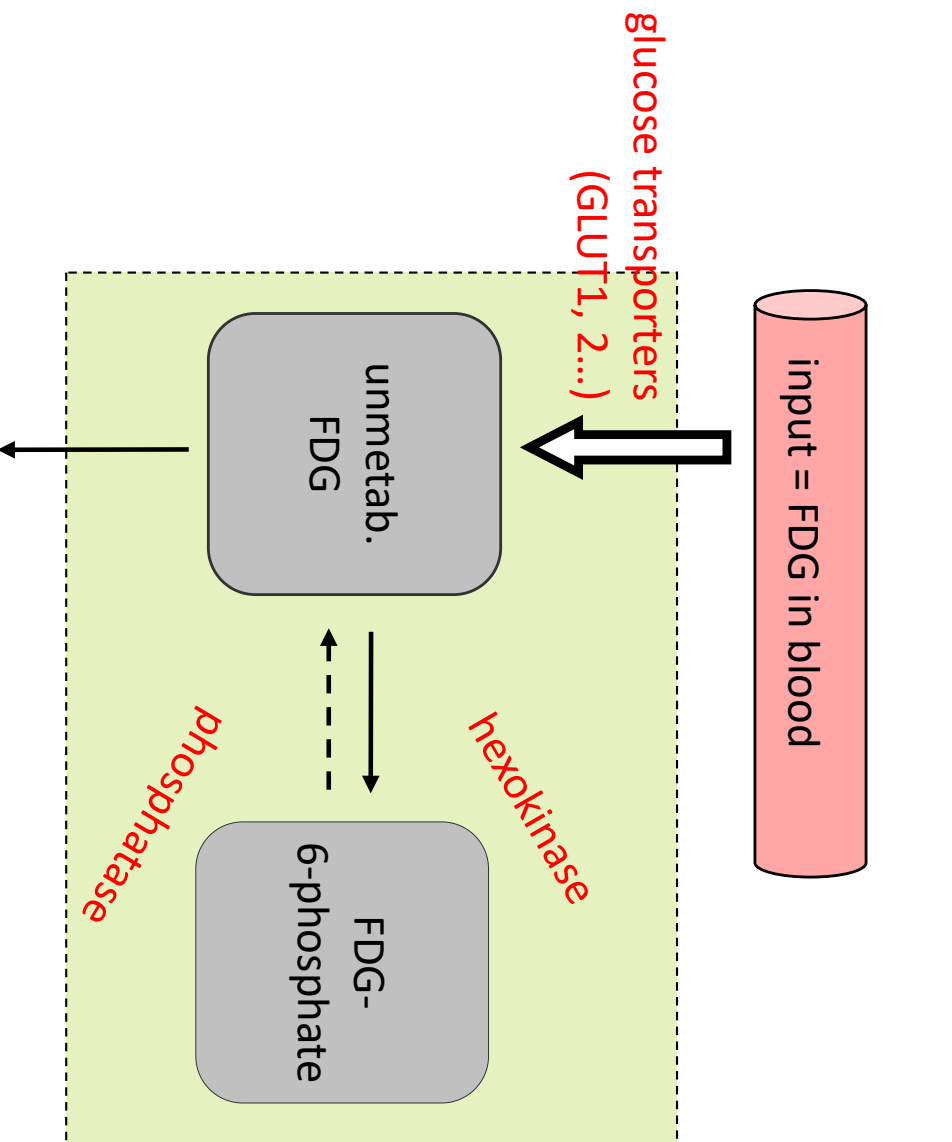
chironomus plumosus larvae

Tracer kinetic modelling: **compartment models**

2-tissue compartment model



Tracer kinetic modelling: **compartment models**



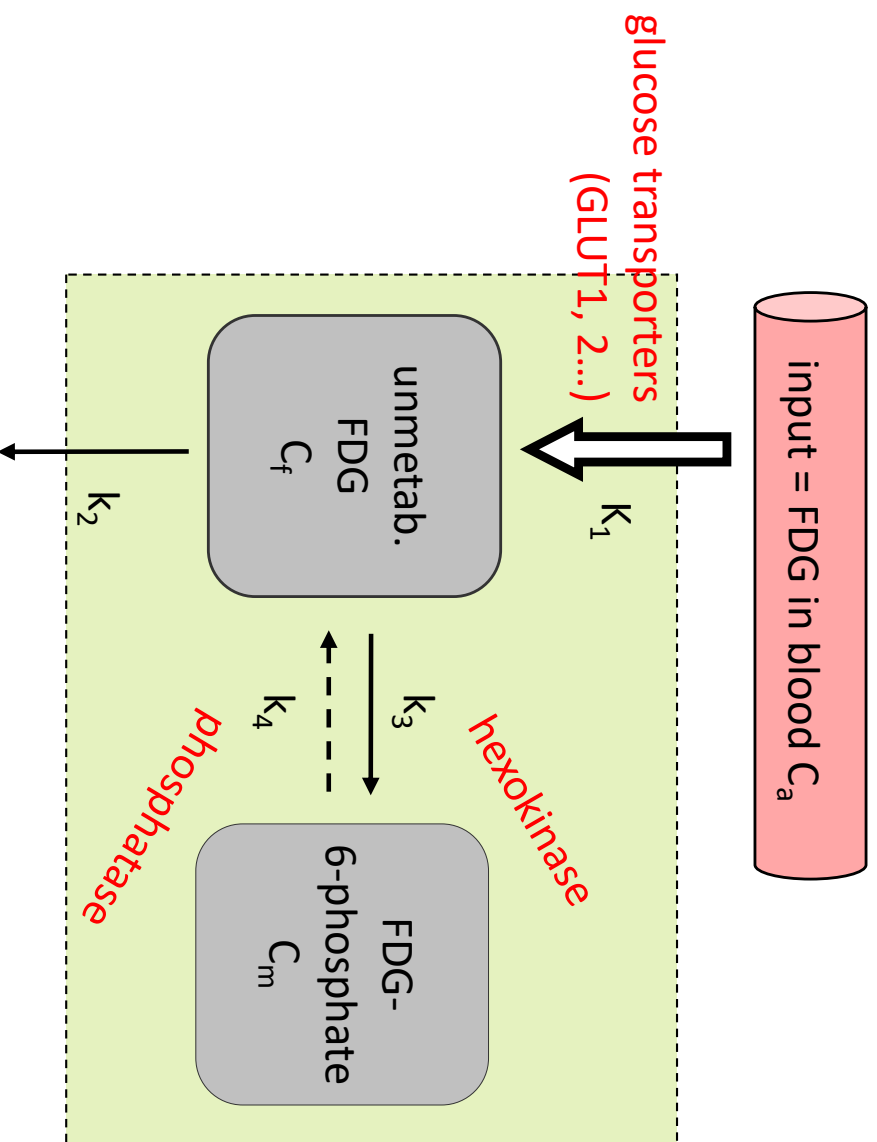
F-18-fluorodeoxyglucose (FDG)

based on known pharmacokinetics*

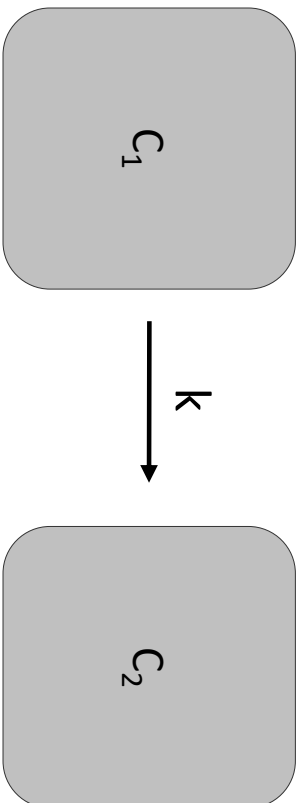
* Bidder TG, Hexose translocation across the blood-brain interface: configurational aspects, J Neurochem 1968, 15: 867-874
Sols A, Crane RK, Substrate specificity of brain hexokinase, J Biol Chem 1954, 210: 581-595
Sokoloff L et al., The [¹⁴C]deoxyglucose method..., J Neurochem 1977, 28: 581-595

Tracer kinetic modelling: compartment models

FDG



Tracer kinetic modelling: translation graphical representation → equations

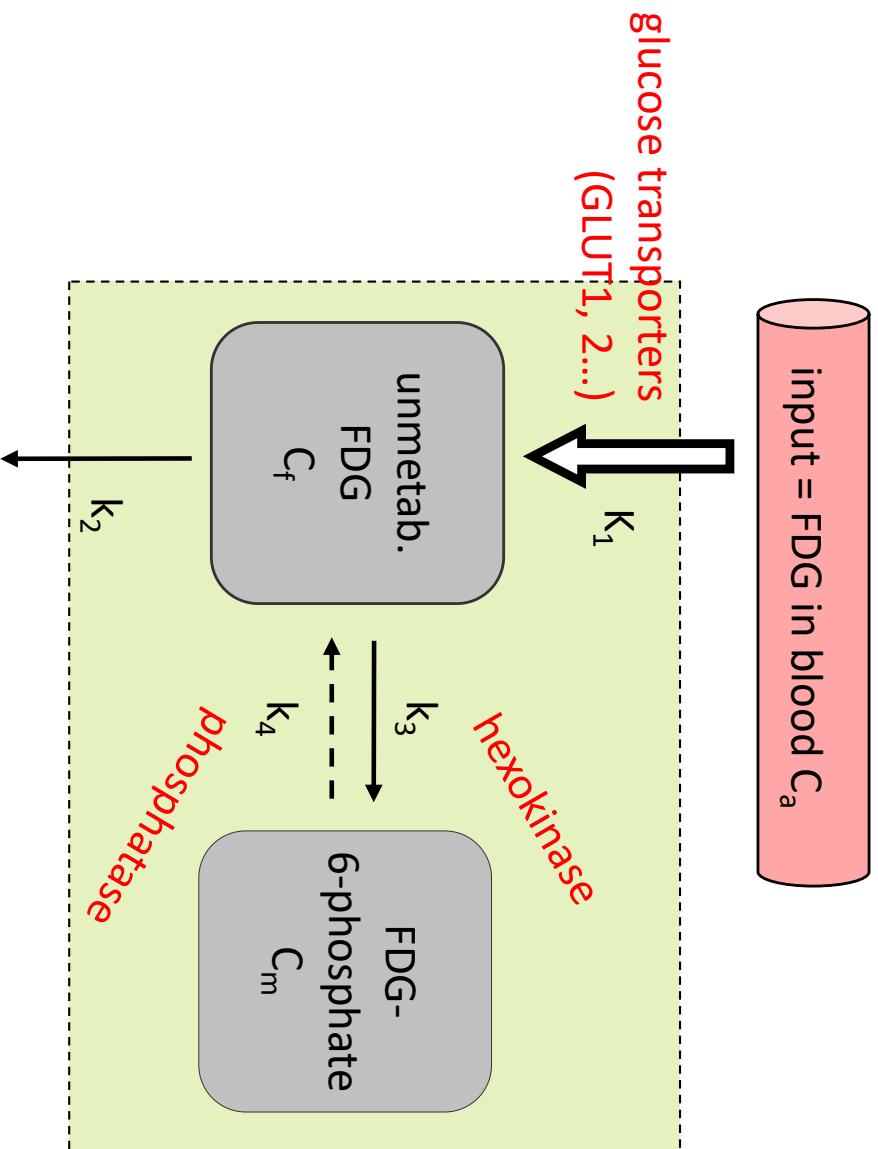


$$dC_1/dt = -k * C_1$$

$$dC_2/dt = k * C_1$$

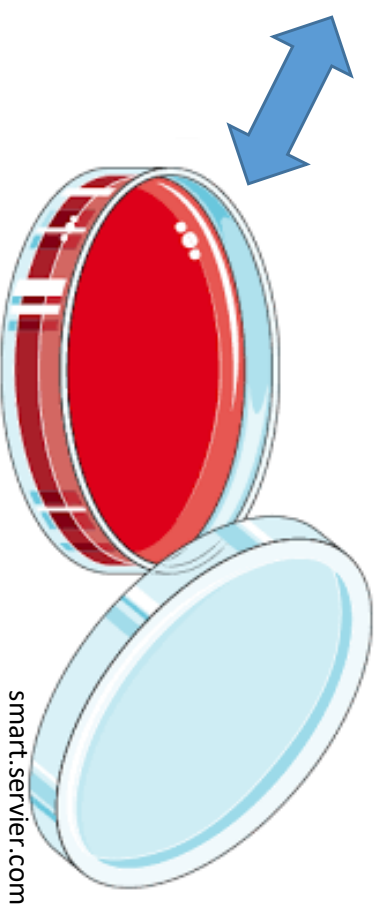
Tracer kinetic modelling: compartment models

FDG



$$dC_f/dt = K_1 * C_a - K_2 * C_f - K_3 * C_f + K_4 * C_m$$

$$dC_m/dt = K_3 * C_f - K_4 * C_m$$

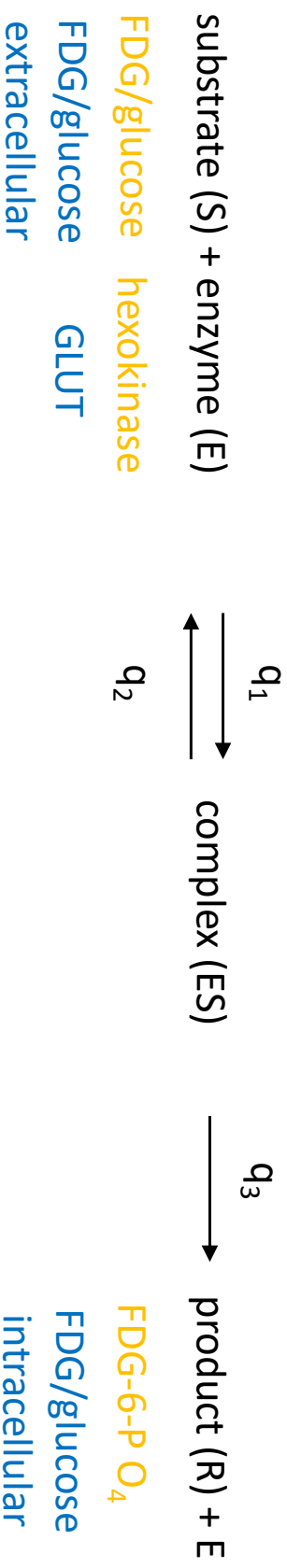


smart.servier.com

Michaelis-Menten model

Michaelis-Menten model of enzyme kinetics (in vitro)

enzyme-mediated reaction



Operational equations

$$(1a) \quad dC_{ES}/dt = q_1 * C_E * C_S - (q_2 + q_3) * C_{ES}$$

$$(1b) \quad V = dC_R/dt = q_3 * C_{ES} \quad \text{reaction rate}$$

$$(1c) \quad \text{total } C_E = C_E + C_{ES}$$

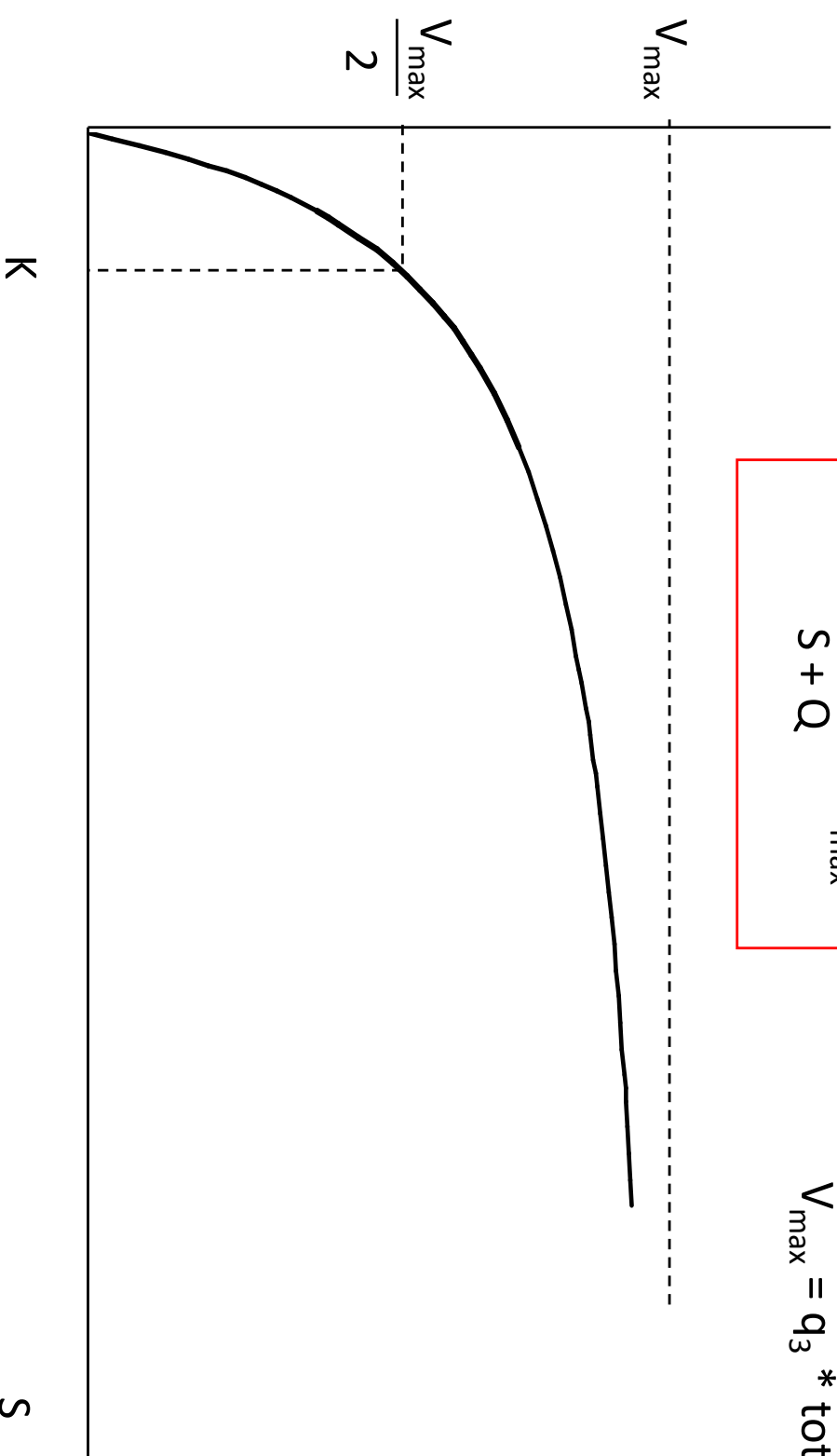
Michaelis-Menten model of enzyme kinetics II

steady state ($dc_{ES}/dt = 0$)

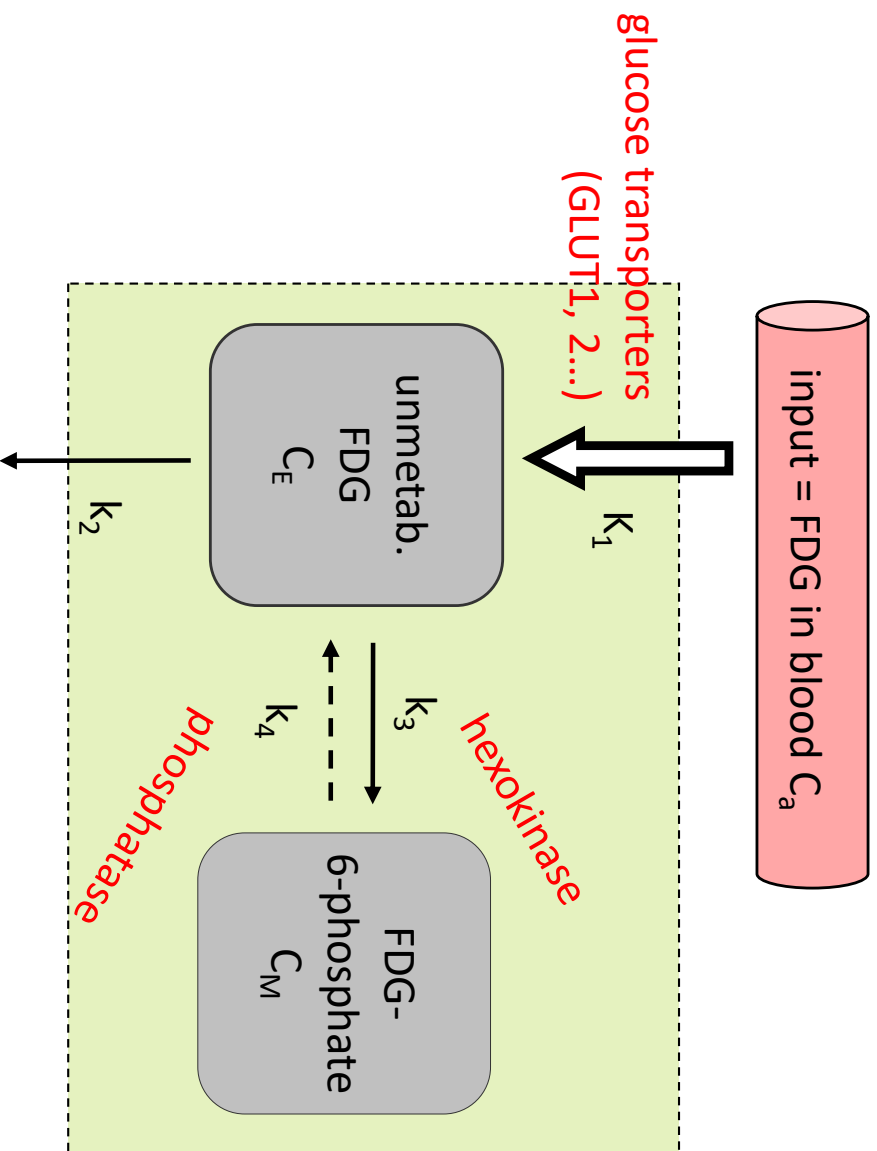
$$V = \frac{S}{S + Q} * V_{\max}$$

$$Q = \frac{q_2 + q_3}{q_1}$$

$$V_{\max} = q_3 * \text{total } E$$



Tracer kinetic modelling: compartment models



compartment Michaelis-

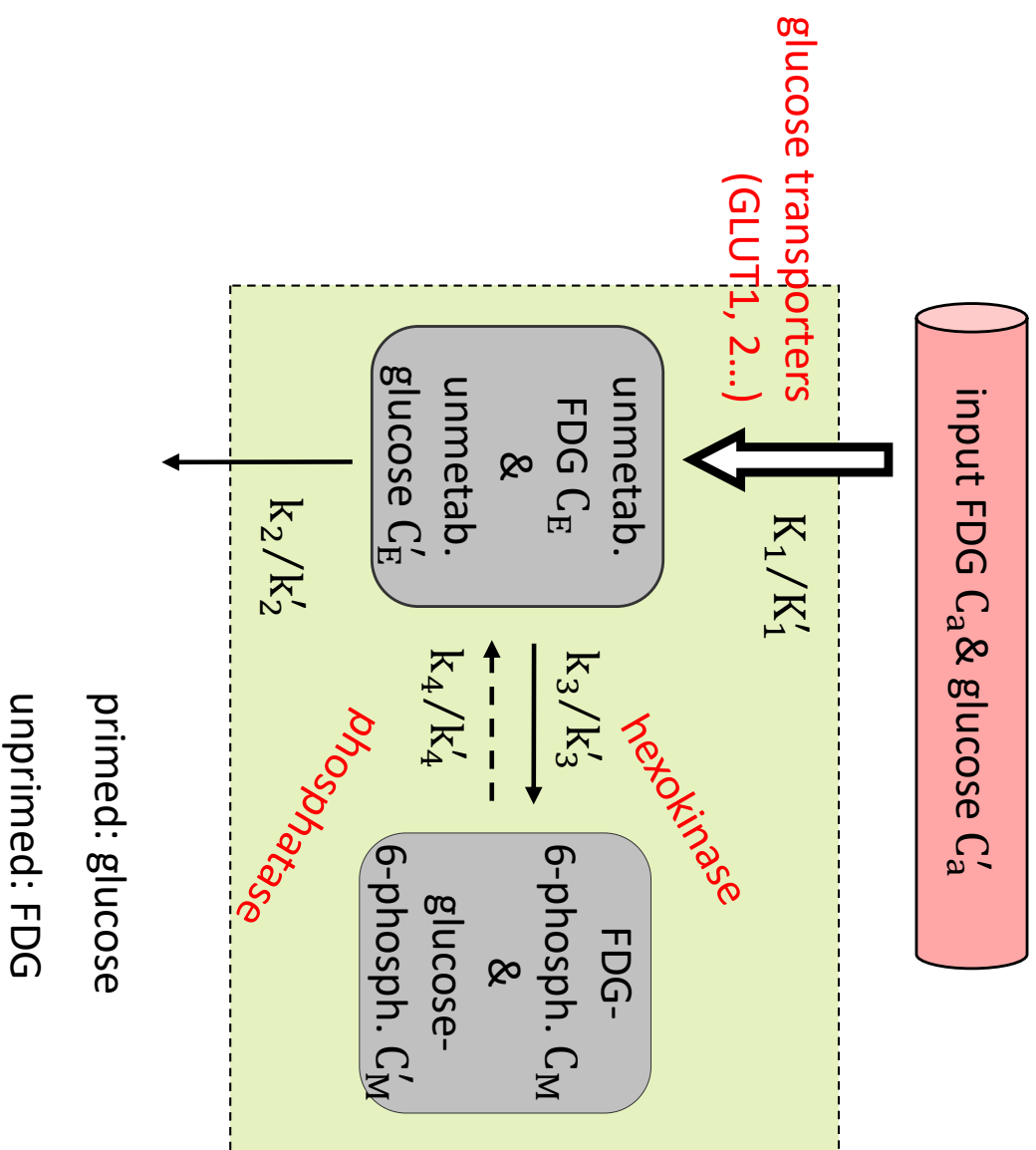
model Menten

transport K_1, k_2 Q_T, V_T

phosphorylation k_3, k_4 Q_M, V_M

Tracer kinetic modelling: competing substrates

Buchert et al., Nuklearmedizin 2009; 48: 44-54



| | compartment | Michaelis- |
|-----------------|-------------|-------------------|
| transport | K_1, k_2 | Menten Q_T, V_T |
| phosphorylation | k_3, k_4 | Q_M, V_M |

$$K_1 = \frac{V_T}{Q_T * (1 + C'_a / Q'_T) + C_a}$$

$$k_3 = \frac{V_M}{Q_M * (1 + C'_E / Q'_M) + C_E}$$

if tracer principle fulfilled, i.e.

$$C_a \ll Q_T * (1 + C'_a / Q'_T)$$

$$C_E \ll Q_M * (1 + C'_E / Q'_M)$$

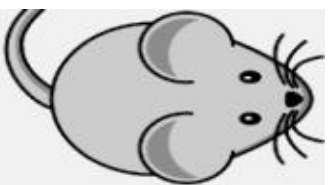
$$K_1 \approx \frac{V_T}{Q_T * (1 + C'_a / Q'_T)}$$

$$k_3 \approx \frac{V_M}{Q_M * (1 + C'_E / Q'_M)}$$

Tracer principle: **Cave!**

1. total tracer, both labelled and unlabelled → specific activity

2.

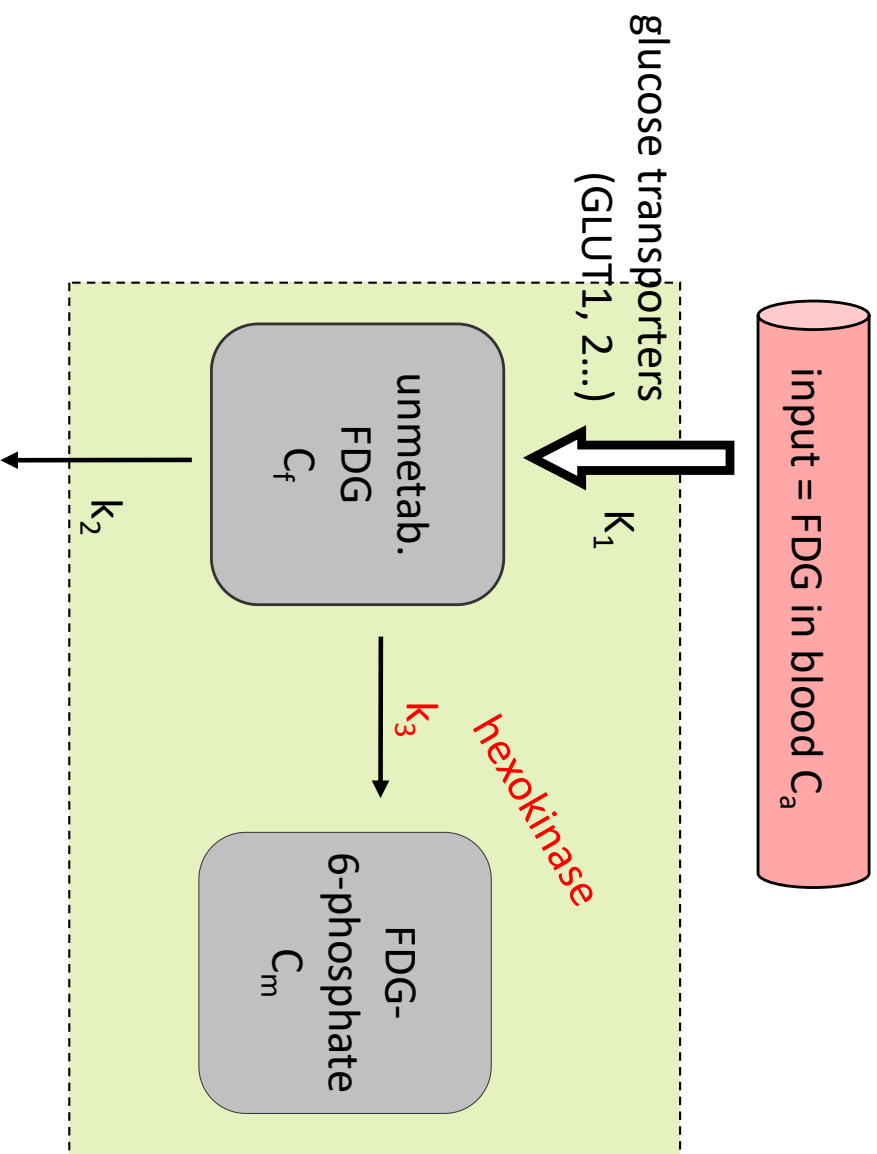


1.5 ml blood



5 L blood

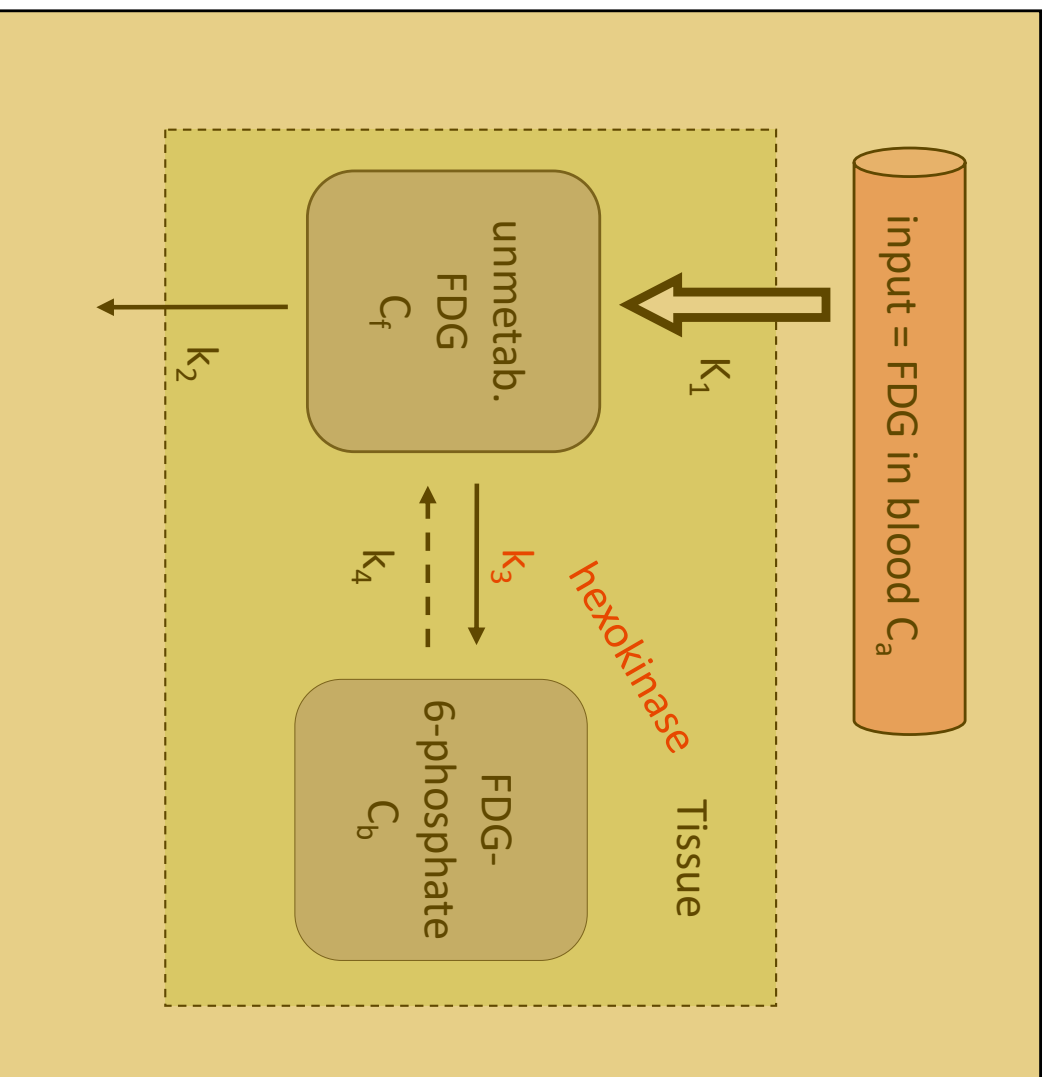
Tracer kinetic modelling: compartment models



$$dC_m/dt = k_3 * C_f$$

$$k_3 = (dC_m/dt) / C_f$$

Tracer kinetic modelling: problem



compartments defined

by function, not localization

PET image volume element

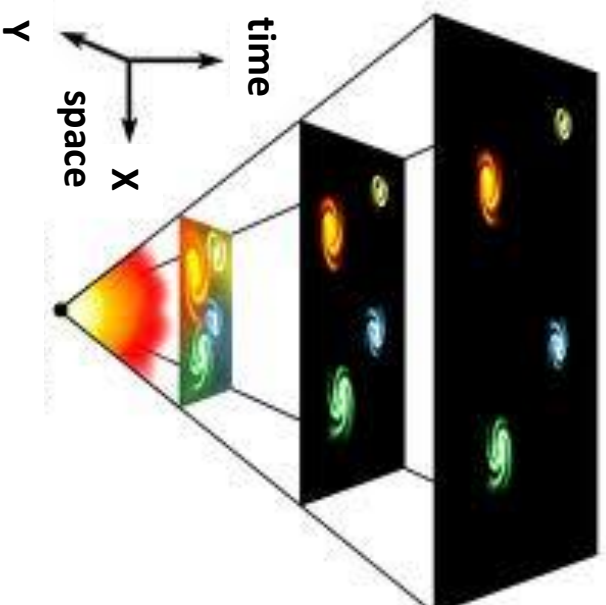
(voxel) =

sum of signals

from all compartments

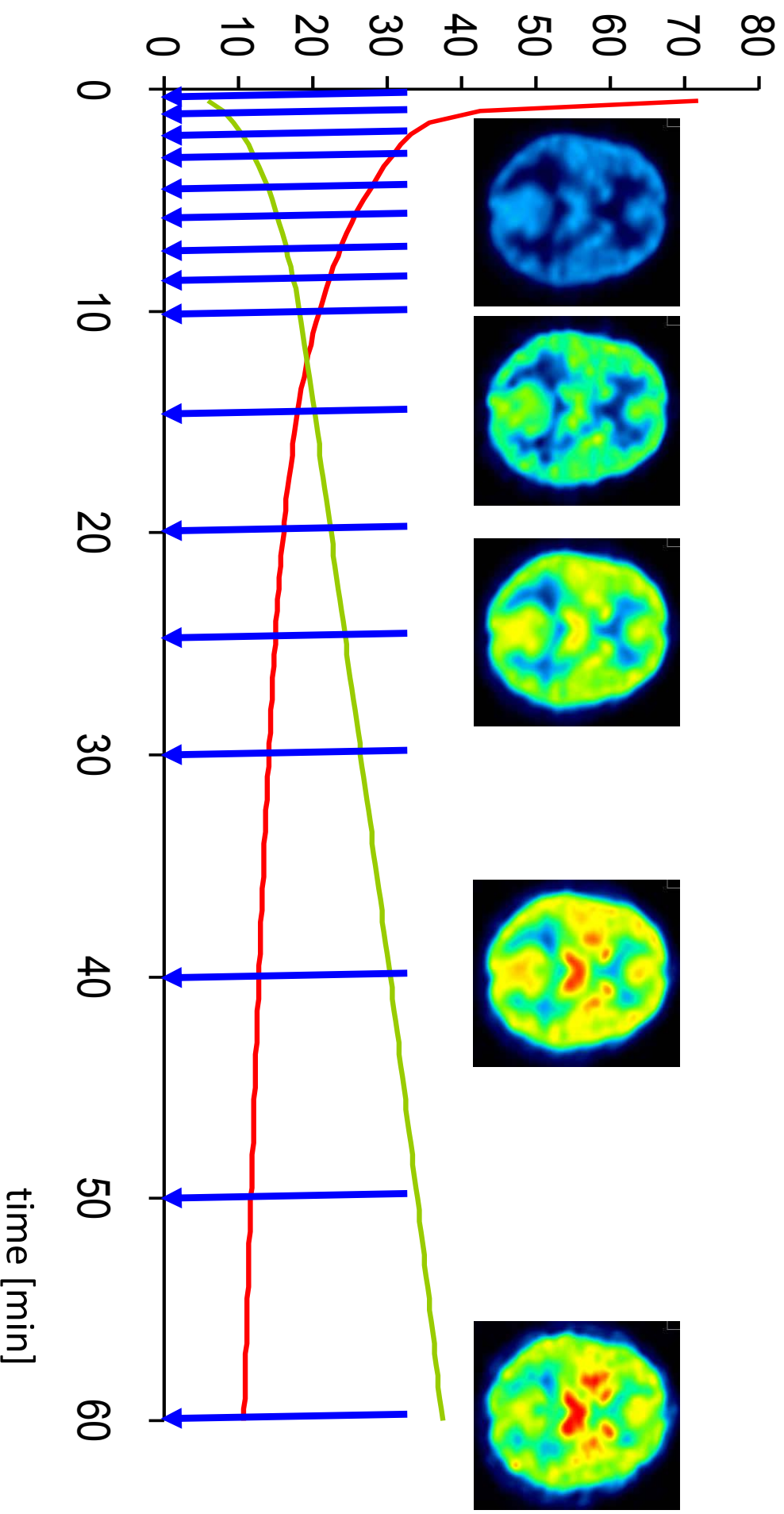
Tracer kinetic modelling: solution

functional compartments might be separated in **time** (rather than space)



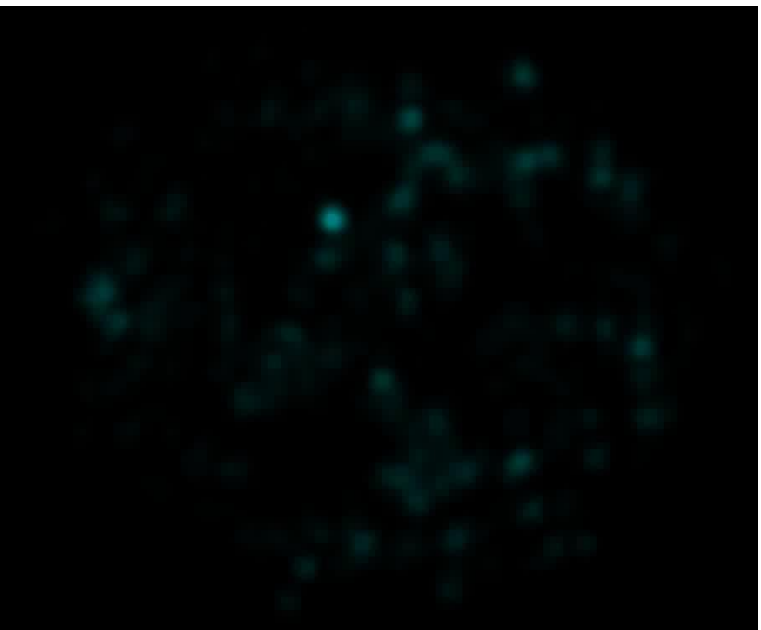
Static imaging → dynamic imaging

activity concentration [kBq/ml]



dynamic imaging

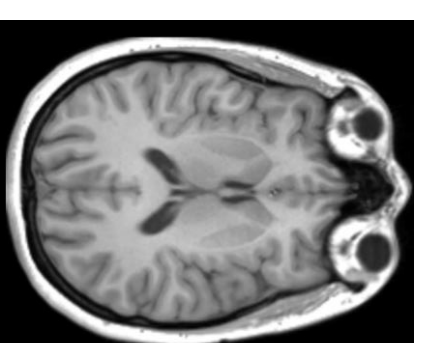
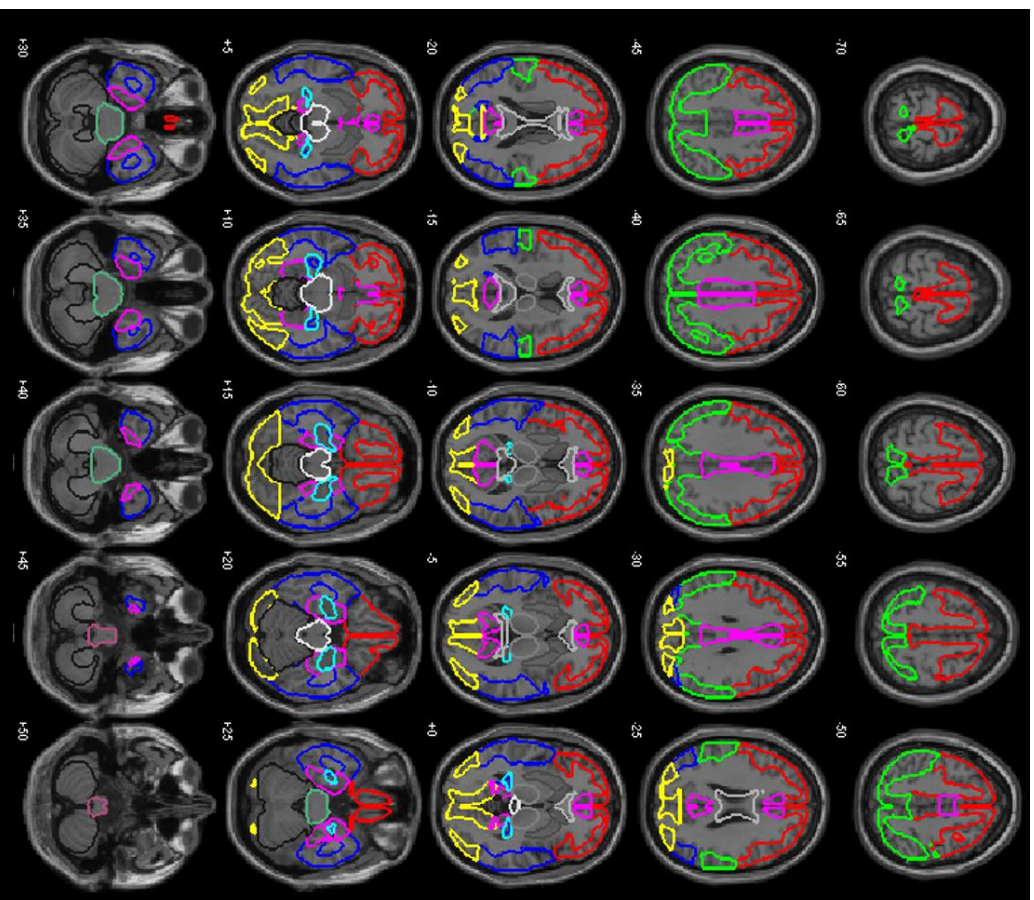
F-18-Fallypride



motion correction!



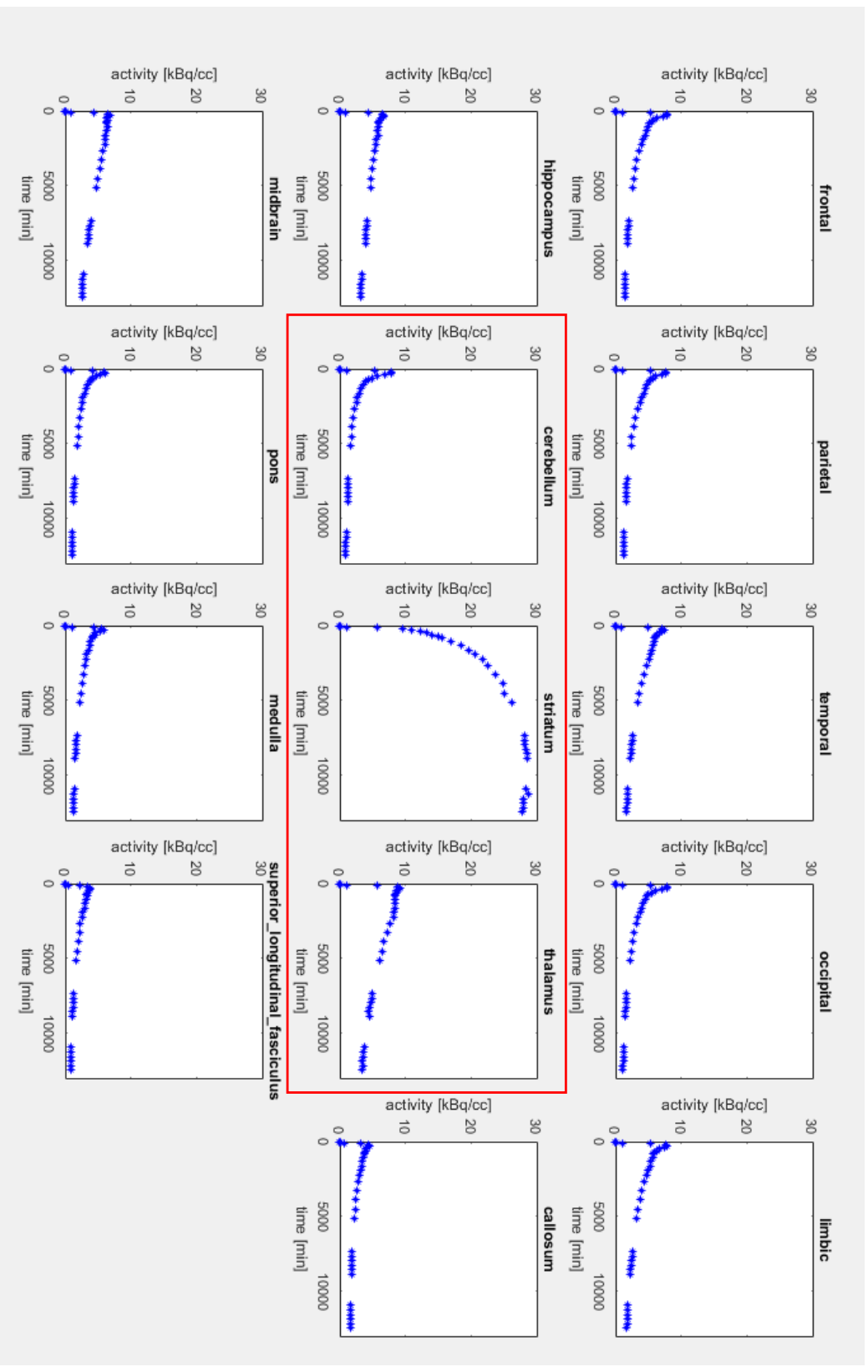
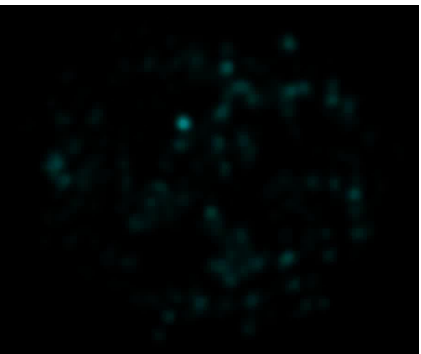
dynamic imaging: region-of-interest (ROI) based time activity curves



stereotactical normalization

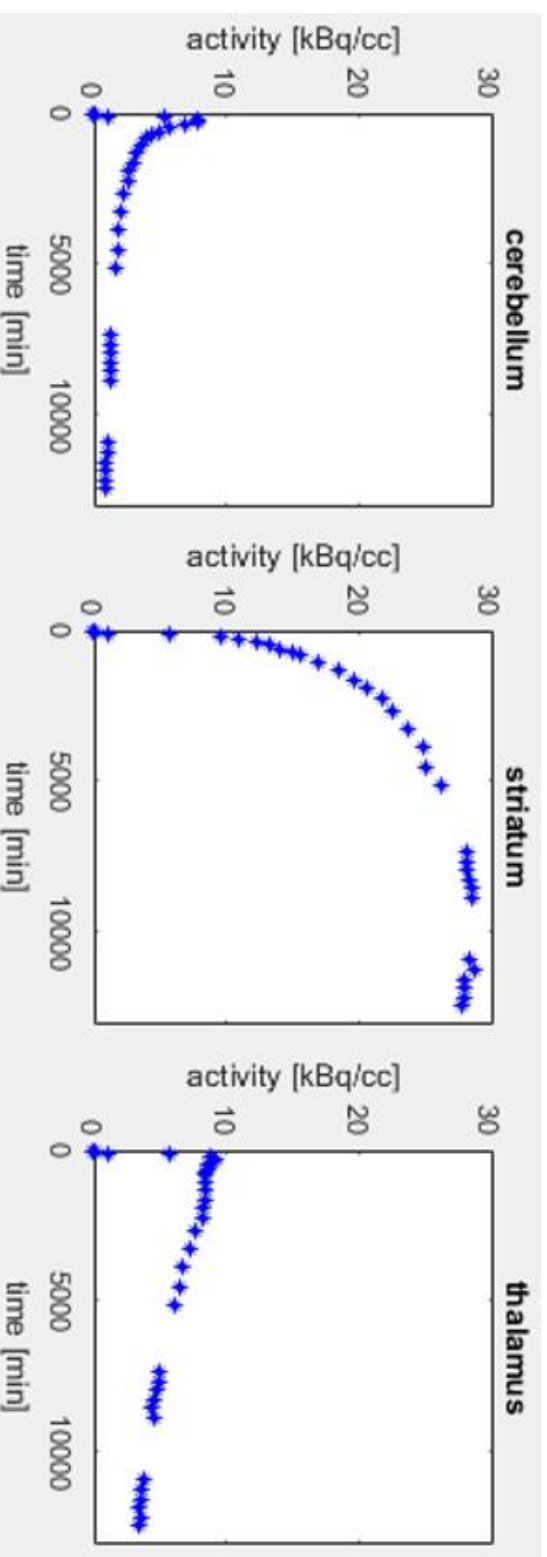
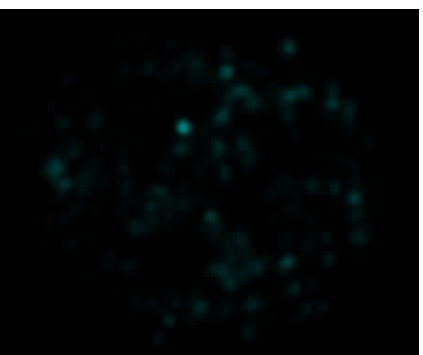
dynamic imaging: region-of-interest (ROI) based time activity curves

F-18-Fallypride



dynamic imaging: region-of-interest (ROI) based time activity curves

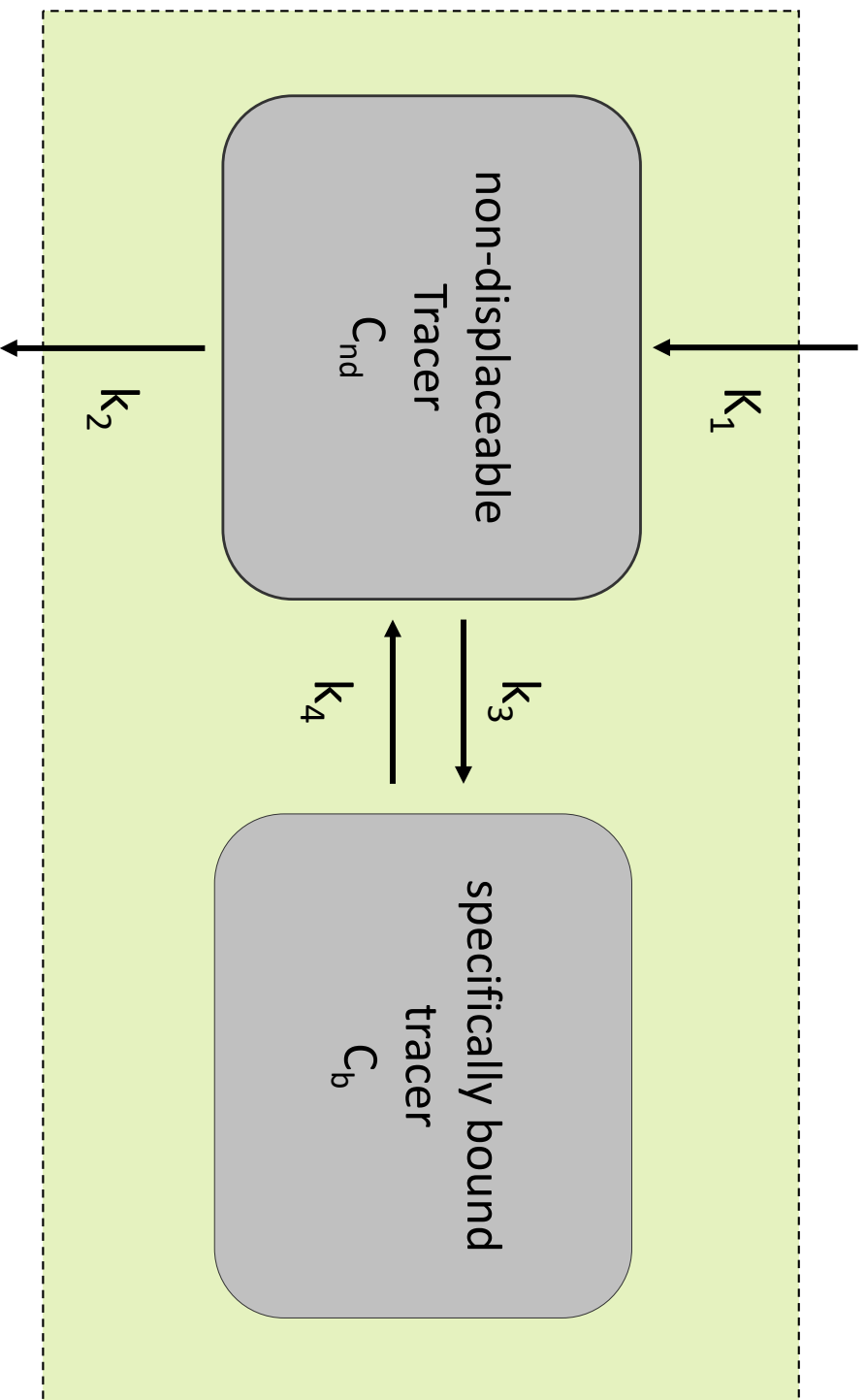
F-18-Fallypride



dynamic imaging: compartment model for F-18-fallypride

mathematically same model as for FDG

input = free fraction of unmetabolized tracer in arterial plasma C_a



$$\frac{dC_{nd}}{dt} = K_1 * C_a - (k_2 + k_3) * C_{nd} + k_4 * C_b$$

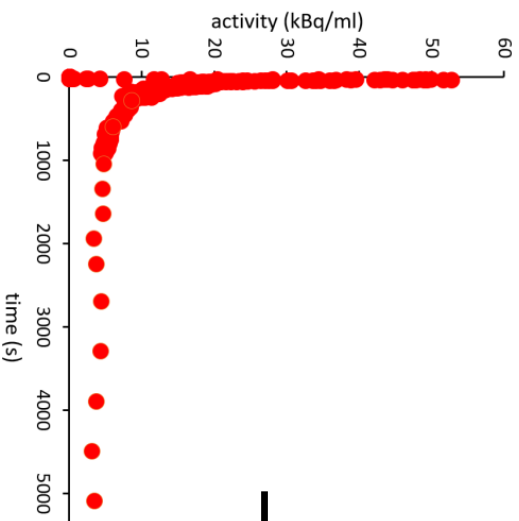
$$\frac{dC_b}{dt} = k_3 * C_{nd} - k_4 * C_b$$

$$C_t = C_{nb} + C_b \quad (+ fbv * C_{blood})$$

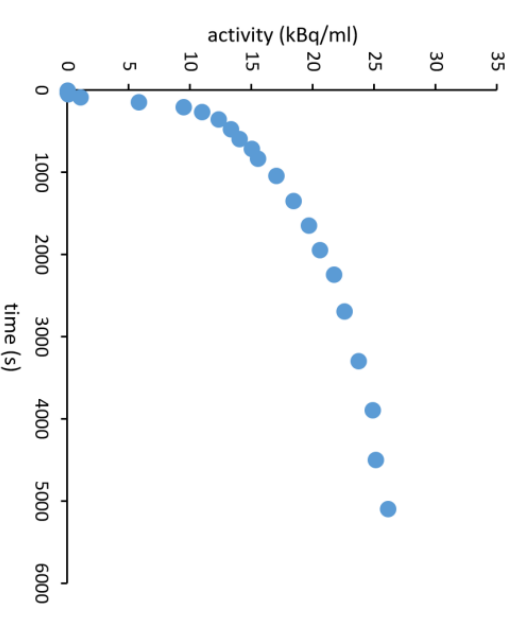
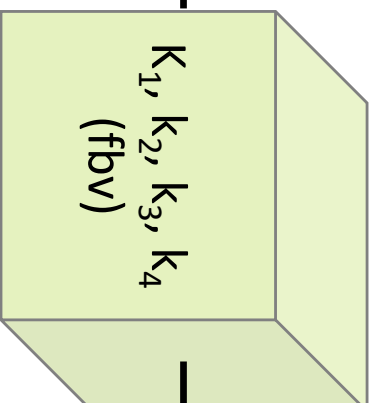
linear system

PET ROI

compartment model



input



Output = C_t

unmetabolized tracer in arterial plasma (t)

linear function(input; $K_1, k_2, k_3, k_4, fbv, t$)

||

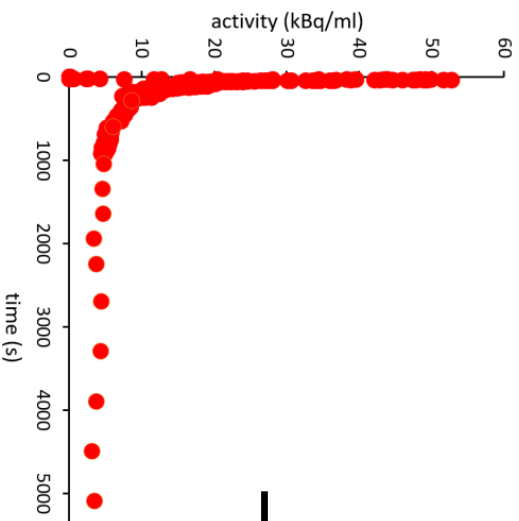
PET time activity curve

noise, systematic errors → no exact solution

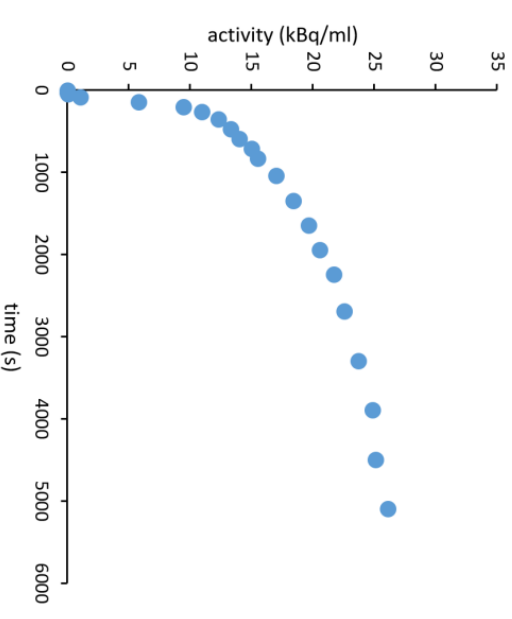
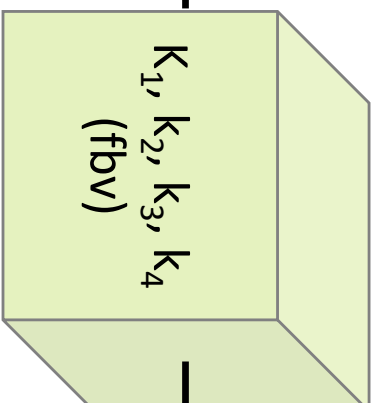
linear system

PET ROI

compartment model



input



output

improved estimate

difference

unmetabolized tracer in arterial plasma (t)

function(input; $K_1, K_2, K_3, K_4, fbv, t$)

PET time activity curve

approximate solution

arterial blood sampling

automatic



manual

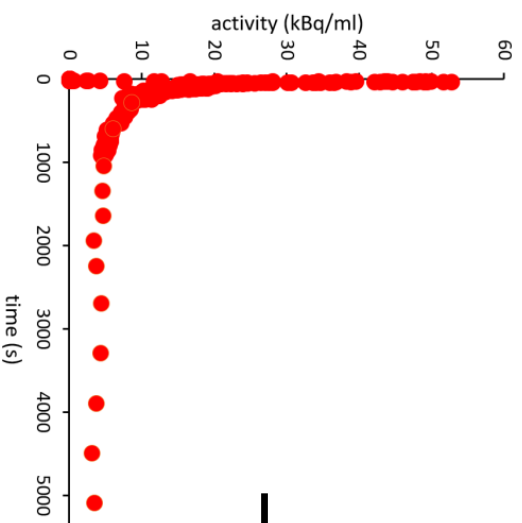


- cross-calibration
- synchronization
- delay
- dispersion

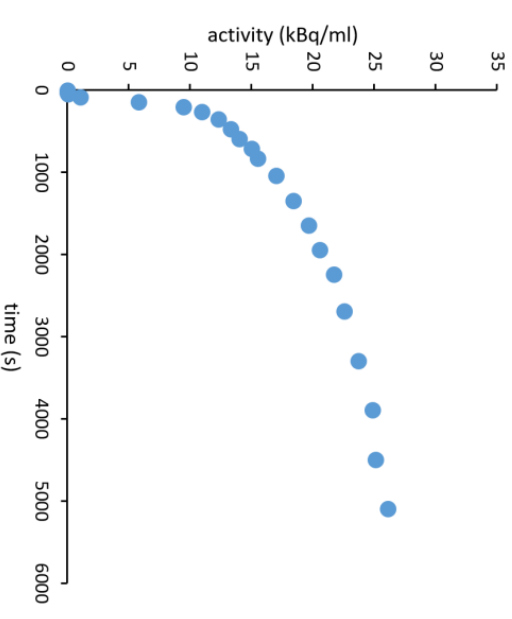
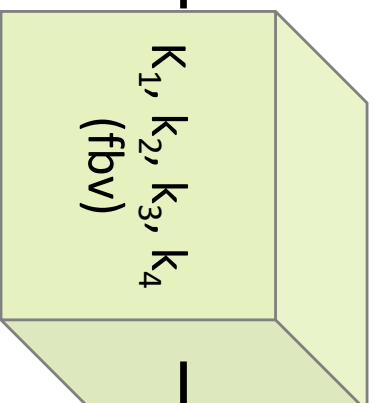
linear system

PET ROI

compartment model



input

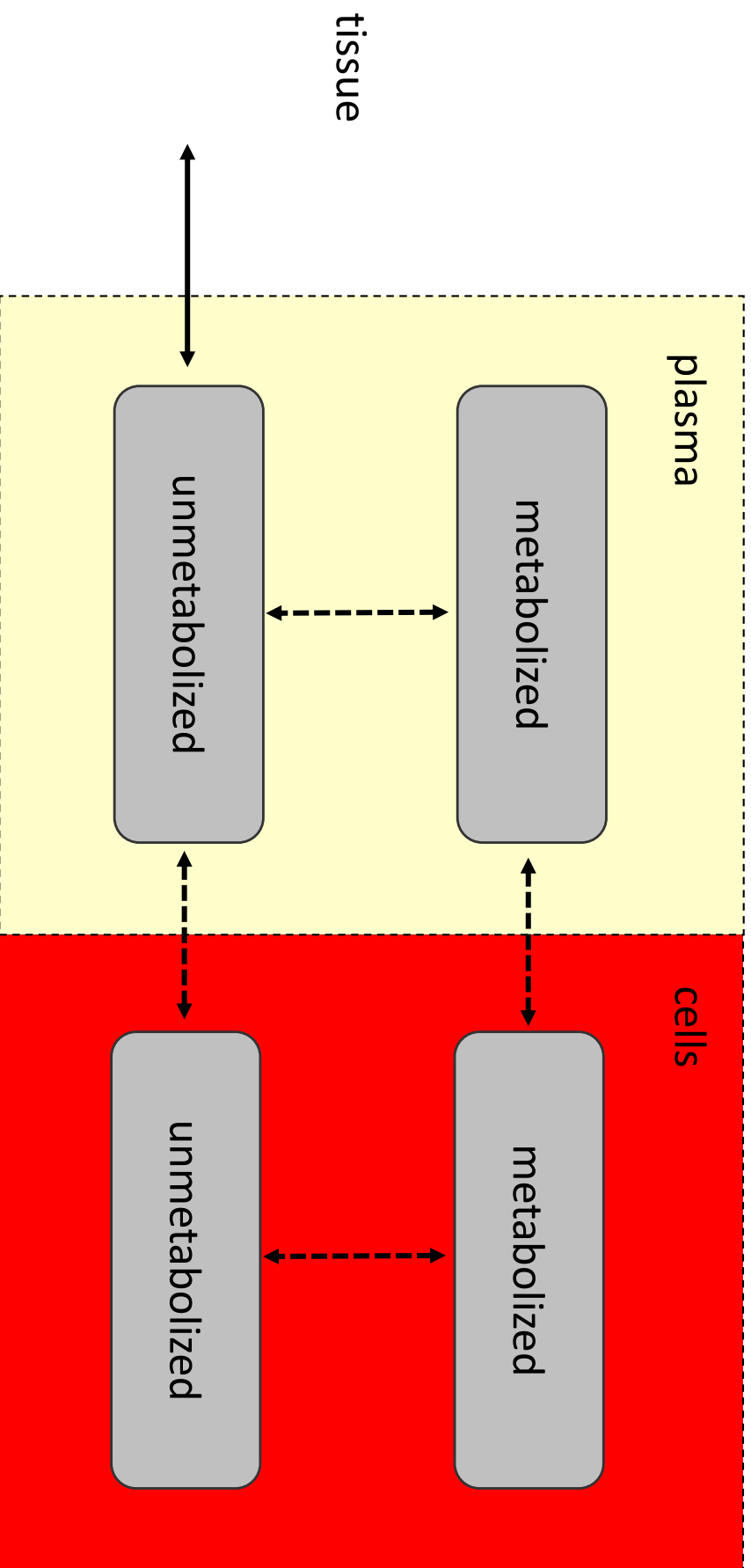


output

unmetabolized tracer in arterial plasma (t)

function(input; $K_1, K_2, K_3, K_4, f_{bv}, t$)

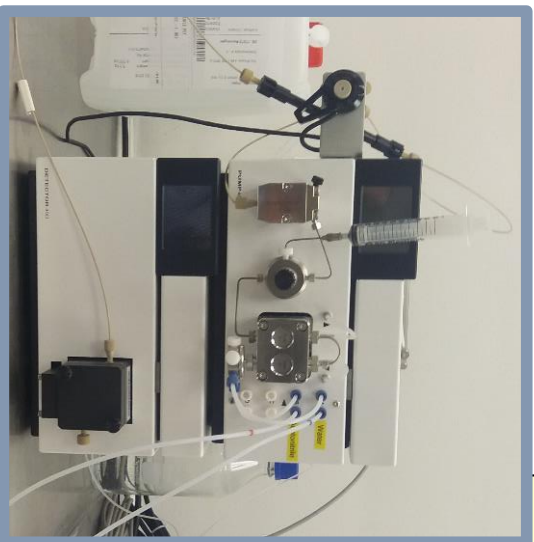
Blood compartment model



neglected

- uptake of metabolized tracer
- binding to plasma proteins

Blood compartment model



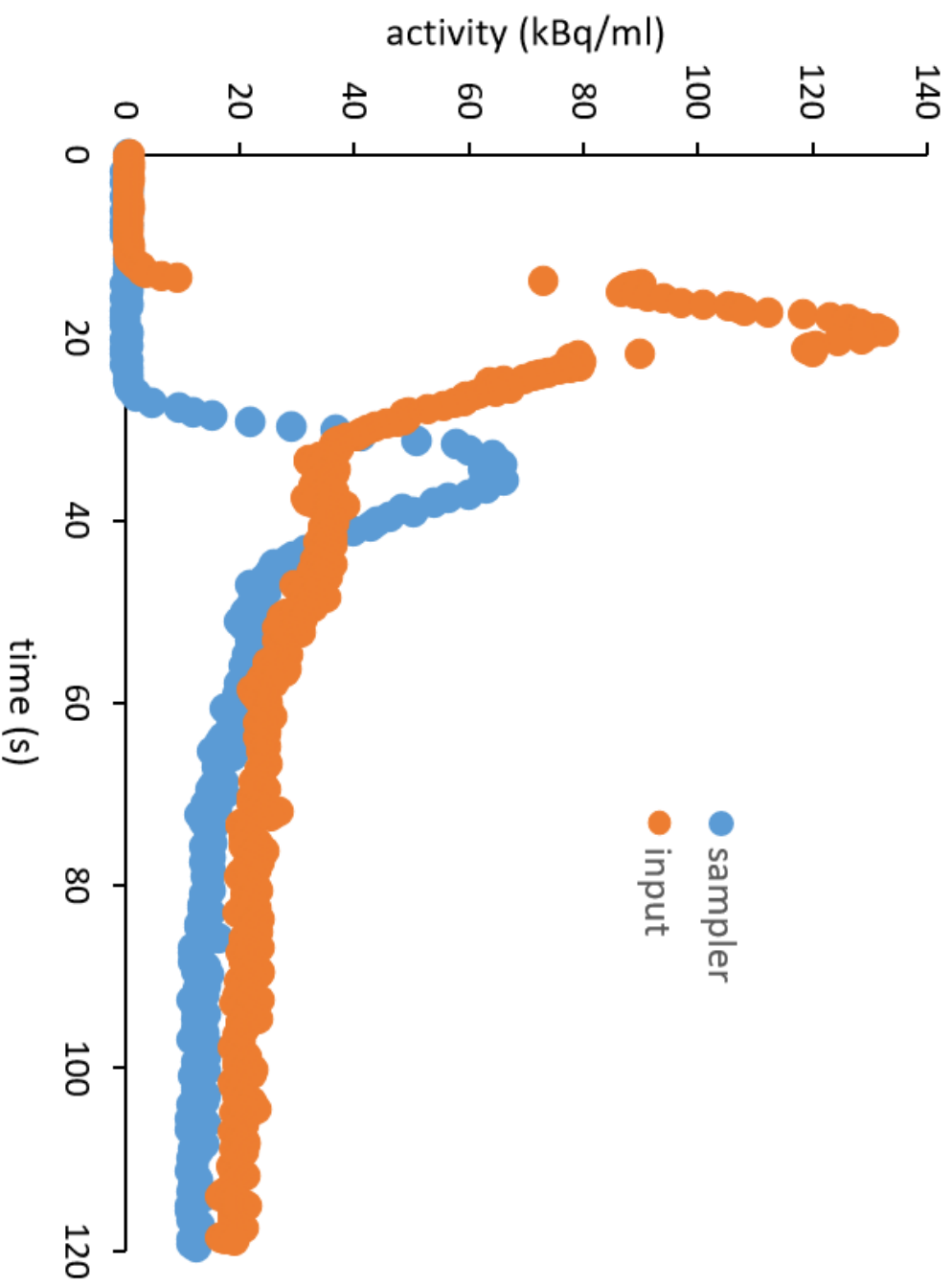
metabolized

unmetabolized

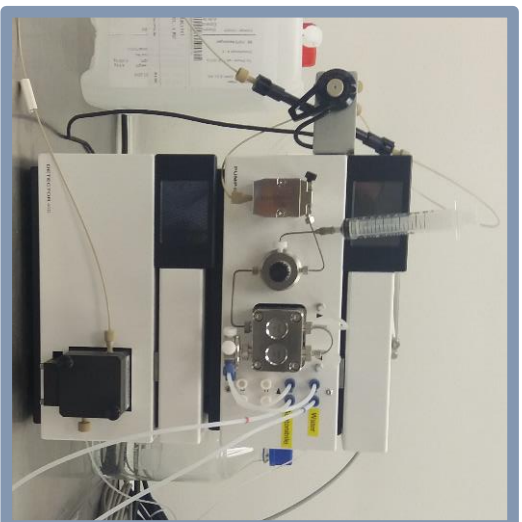


cell fraction

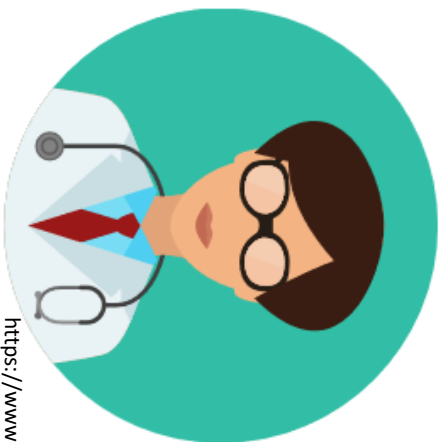
Input function



Tracer kinetic modelling: technical requirements

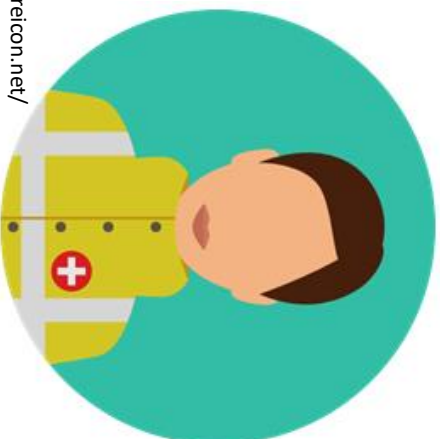


Tracer kinetic modelling: **staff**

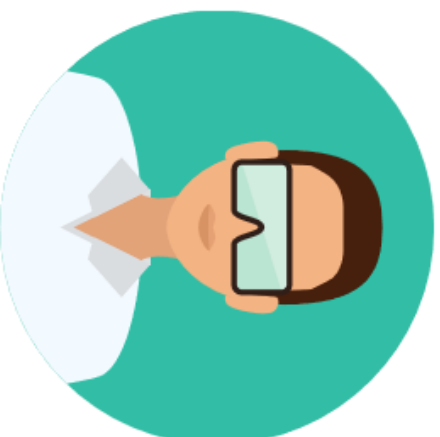


<https://www.shareicon.net/>

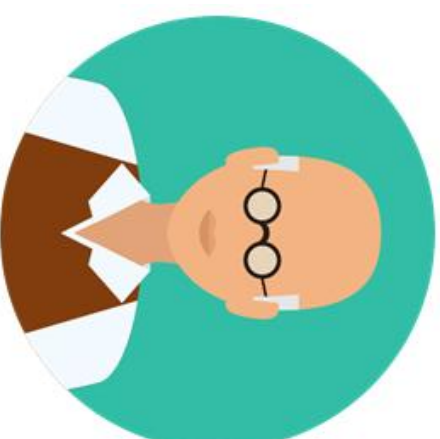
physician



technician



radiochemist



physicist



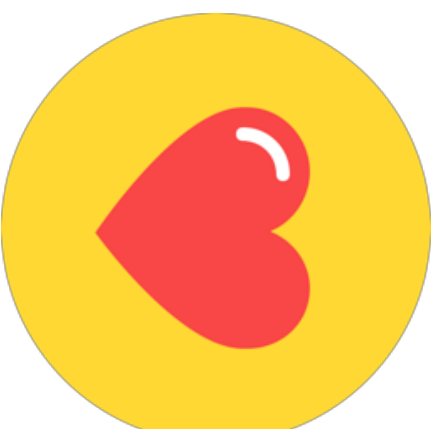
<http://www.zarata.info>

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<http://news.softpedia.com>

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Contents

- basic principle of PET and SPECT radionuclide imaging & tracer principle
- tracer kinetic modelling
 - application in medical research
 - how it works



chironomus plumosus larvae

Chironomus plumosus larvae

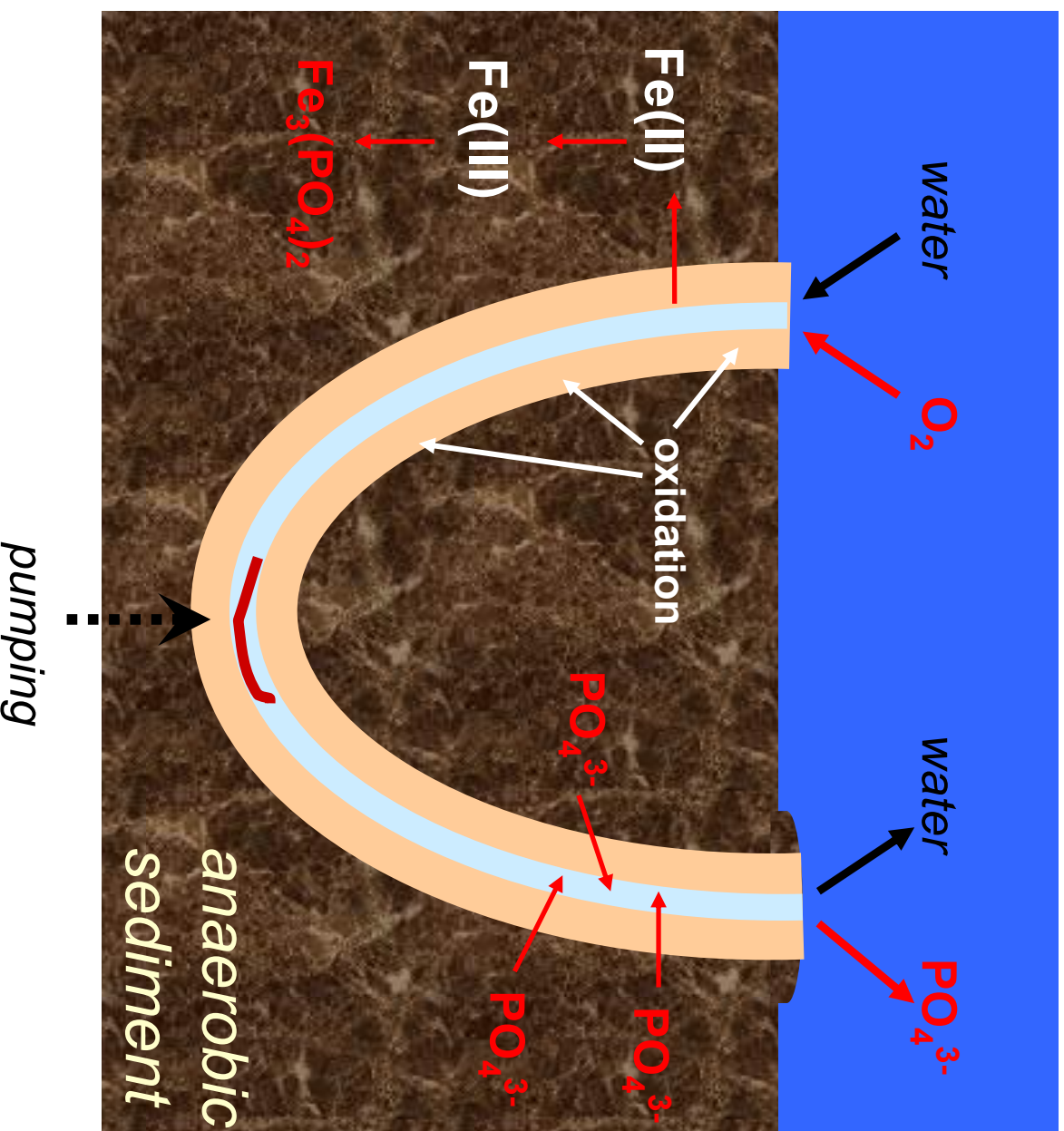


up to 10.000 per m² lake sediment



building u-shaped burrows (15 cm deep) inside lake sediments

Chironomus plumosus larvae

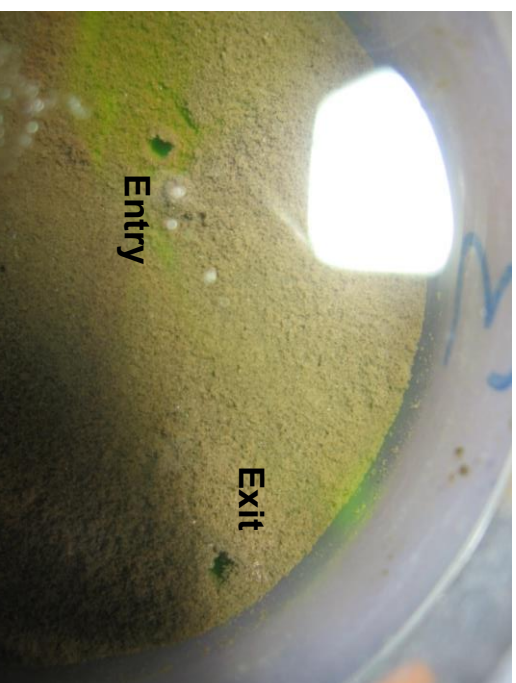


genesis of
phosphorus
iron complex
↓
**phosphorus
retention**
in the
sediment

**phosphorus
removal**
to water body
via
convective
transport

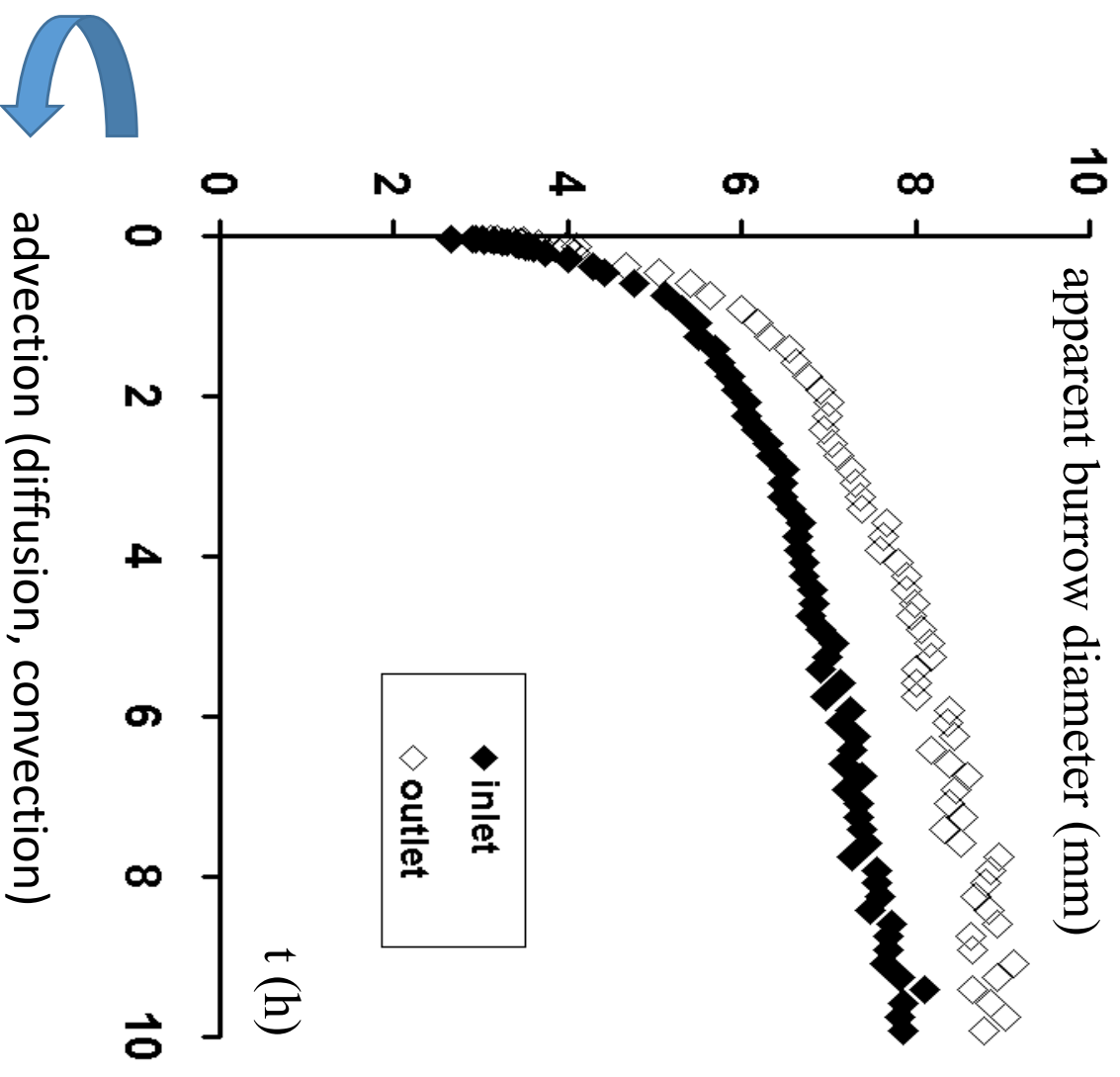
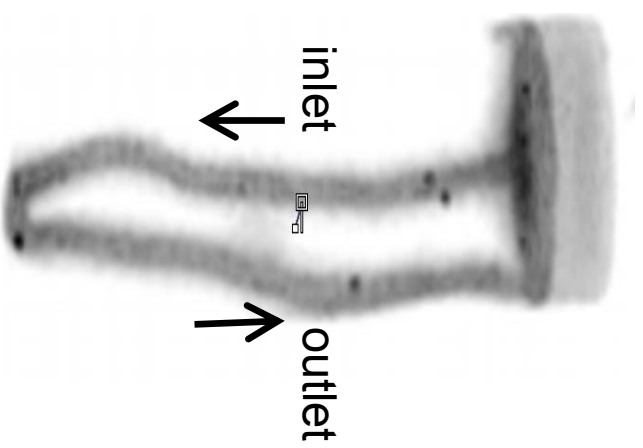
Chironomus plumosus larvae: dynamic small animal PET

Roskosch A et al., Appl Radiat Isot 2010; 68:1094-7.



Chironomus plumosus larvae: dynamic small animal PET

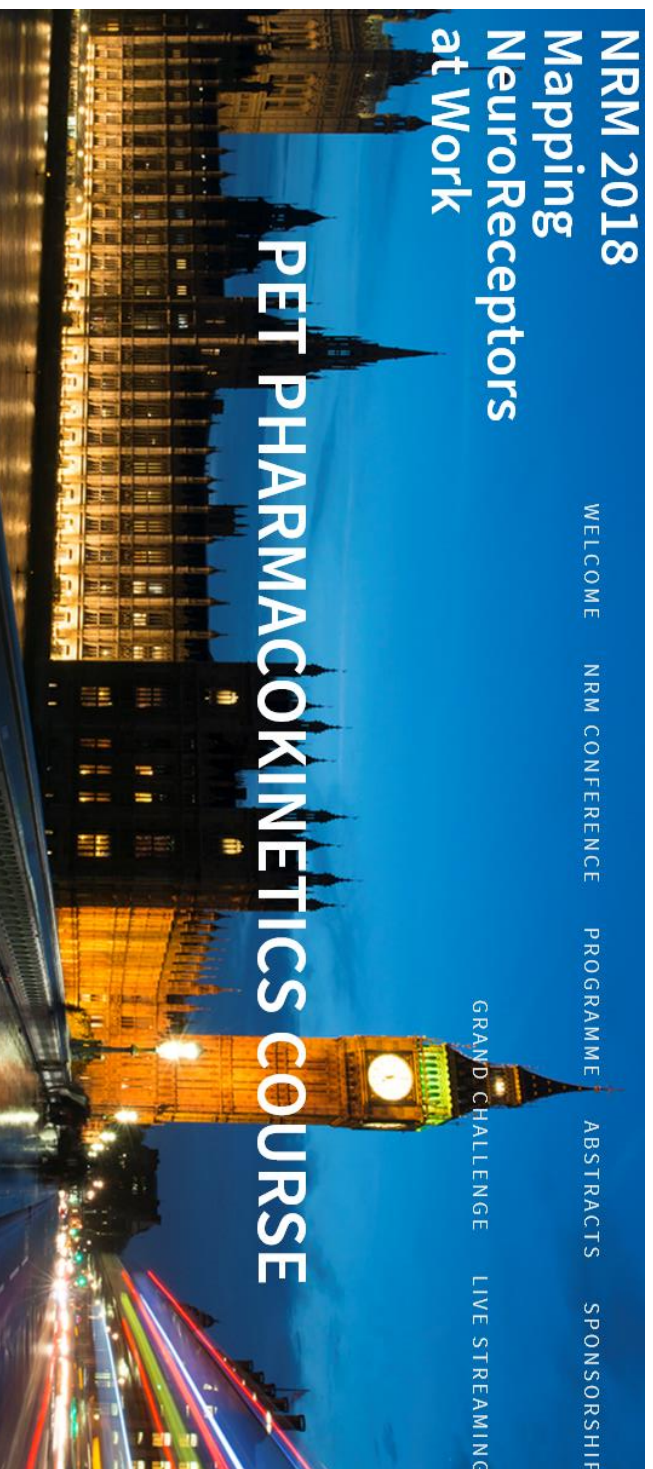
Roskosch A et al., Appl Radiat Isot 2010; 68:1094-7.



advection (diffusion, convection)

Further reading

- Carson R. Tracer Kinetic Modeling in PET, in Valk et al., Positron Emission Tomography: Basic Science and Clinical Practice
- Slifstein M, [Laruelle M](#). Models and methods for derivation of in vivo neuroreceptor parameters with PET and SPECT reversible radiotracers. [Nucl Med Biol](#). 2001, 28:595-608
- Laruelle M. Imaging synaptic neurotransmission with in vivo binding competition techniques: a critical review. [J Cereb Blood Flow Metab](#). 2000, 20:423-51
- Gunn RN, [Gunn SR](#), [Cunningham VJ](#). Positron emission tomography compartmental models. [J Cereb Blood Flow Metab](#). 2001, 21: 635-52
- van den Hoff J. Principles of quantitative positron emission tomography, Amino Acids 2005, 29: 341-353
- Innis RB et al. Consensus nomenclature for in vivo imaging of reversibly binding radioligands, JCBFM 2007, 27: 1533-1539



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